Analytical Explanations

Physics

1. (*d*) **Idea** The elastic potential energy stored in wire is given by

$$E = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

and $\text{stress} = Y(\text{strain})$

Elastic energy

$$E = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2} \times \frac{(\text{stress})^2}{Y} \times \text{volume}$$

Volumes of three wires are

$$V_1 = (\pi R^2)\, L = \pi R^2 L$$

$$V_2 = \pi\,(2R)^2\,(2L) = 8\,\pi R^2 L$$

$$V_3 = \pi \times (3R)^2 \times (3L) = 27\,\pi R^2 L$$

Streses of three wires are

$$\Rightarrow \quad \sigma_1 = \frac{w}{\text{Area}} = \frac{w}{\pi R^2}$$

$$\sigma_2 = \frac{w}{4\pi R^2} \text{ and } \sigma_3 = \frac{w}{9\pi R^2}$$

Now, $E = E_1 + E_2 + E_3$

$$\Rightarrow \quad E = \frac{1}{2} \cdot \frac{\sigma_1^2}{Y} \cdot V_1 + \frac{1}{2} \frac{\sigma_2^{\,2}}{Y} \cdot V_2 + \frac{1}{2} \cdot \frac{\sigma_3^2}{Y} \cdot V_3$$

$$= \frac{1}{2}\left[\frac{w}{\pi R^2}\right]^2 \cdot \frac{\pi R^2 L}{Y} + \frac{1}{2}\left[\frac{w}{4\pi R^2}\right]^2 \cdot \frac{8\pi R^2 L}{Y}$$

$$+ \frac{1}{2}\left[\frac{w}{9\pi R^2}\right]^2 \frac{27}{Y} \times \pi R^2 L$$

$$\Rightarrow \quad E = \frac{w^2 L}{2\pi R^2 Y} + \frac{w^2 L}{4\pi R^2 Y} + \frac{w^2 L}{6\pi R^2 Y} = \frac{11 w^2 L}{12\pi R^2 Y}$$

TEST Edge Question related to stress, strain, Young's modulus, energy stored in wire and work done in stretching wire etc., are frequently asked. Students are advised to go through these concepts deeply and also memorise important formulae.

2. (*b*) **Idea** The radiant power *i.e.*, energy radiated by a body per unit time is given by

$$\frac{Q}{t} = A\varepsilon\sigma T^4$$

$$\Rightarrow \quad Q \propto AT^4$$

also $\lambda_m T = \text{constant}$

$$\Rightarrow \quad Q \propto \frac{A}{(\lambda_m)^4}$$

Power radiated, $Q \propto AT^4$ and $\lambda_m T = \text{constant}$

Hence, $Q \propto \dfrac{A}{(\lambda_m)^4}$ or $Q \propto \dfrac{r^2}{(\lambda_m)^4}$

$$Q_A : Q_B : Q_C = \frac{(2)^2}{(3)^4} ; \frac{(4)^2}{(4)^4} : \frac{(6)^2}{(5)^4} = 0.05 : 0.0625 : 0.0576$$

i.e., Q_B is maximum.

TEST Edge Question related to Kirchhoff's law, Stefan's law, Newton's law of cooling and Wien's displacement law are frequently asked, students are advised to have better understanding of concept and application of these topics.

3. (*b*) **Idea** When a charge or charged body brought near to neutral conducting body then due to induction some charge of opposite nature will appear in neutral body due to induction.

Suppose the charge is positive q then if it is brought closer to a conducting sphere it will induce charge in it, whose distribution would look something like this.

Although sphere as a whole would have zero charge but due to induction negative charge will be closer to the + q as compared to positively induced charge and hence d would decrease. Same explanation can be given, if q is negative.

TEST Edge Question related to conduction, induction are asked in examination in which nature of charge, amount of charge etc., are asked. Students are advised to understand these concepts properly to solve problems.

4. (*a*) **Idea** Here, particle is moving in two-dimensional $(x - y)$ plane, with constant speed so slope of graph $\dfrac{dy}{dx} = \dfrac{v_y}{v_x}$ and $v_0 = \sqrt{v_x^2 + v_y^2}$.

As slope, $\dfrac{dy}{dx} = -\dfrac{1}{\sqrt{3}} = \dfrac{v_y}{v_x}$

So, $v_x = -v_y\sqrt{3}$

On squaring and adding

$$v_x^2 + v_y^2 = v_0^2$$

$$3v_y^2 + v_y^2 = v_0^2$$

$$v_y = \pm\frac{v_0}{2}$$

$$v_x = \mp\frac{\sqrt{3}v_0}{2}$$

So, $y = \alpha e^{\frac{-2}{\sqrt{3}\,v_0} \times \frac{\sqrt{3}\,v_0}{2}}$

$$y = \alpha e^{-1}$$

TEST Edge Generally, question in the term of graph are frequently asked, *e.g.*, *x*-*T* graph, *v*-*T* graph, etc. So, student must study all sort of graph and must know basic concepts like slope of *x*-*T* graph will give instantaneous velocity etc.

5. (*d*) **Idea** Energy of electron in ground state is
= – (ionisation energy)
If *n*, is the value of excited state, then
number of spectral lines $= \frac{n(n-1)}{2}$

Energy of electron in ground state
= – (ionisation energy)
= – 13.6 eV.
Final energy = – 13.6 + 12.75
= – 0.85 eV

If *n* be the value of excited state, then

$$-\frac{13.6}{n^2} = -0.85$$

$\Rightarrow \quad n^2 = 16$

$\Rightarrow \quad n = 4$

Now, the number of spectral lines are

$$= \frac{n(n-1)}{2} = \frac{4\times 3}{2} = 6 \text{ lines}$$

TEST Edge Energy in Bohr's orbit are frequently asked it can also be asked that how many spectral lines are emitted.

6. (*b*) **Idea** If there is no external force acting then lienar momentum of system will be conserved.

From conservation of momentum theorem, the *X* component of momentum is conserved.

So, $m_1v_1 = m_2v_2$

$\Rightarrow$ *R* will not depend on the vertical height *h*.

if $m_1 > m_2$

Then, $v_1 < v_2$

So, $x_1 < x_2$

The CM of the system of two balls will lie on the same vertical line. It will come down vertically due to gravitational force but it will not move horizontally as $(F_{ext})_x = 0$

TEST Edge Question based on linear momentum conservation, angular momentum conservation, torque etc., are frequently asked in examination. Students are advised to understand these concepts and their applications.

7. (*d*) **Idea** As switch will be shifted then capacitors will not draw more charge from circuit and total charge on 1 μF capacitor will be shared by other capacitors according to their capacity.

Initially, $Q_1 = 24\,\mu C$ $[\because Q = CV]$

Finally, $V_{CQ} = \frac{24}{\frac{6}{5}+1} = \frac{24\times 5}{11}$ V

$$Q_3' = Q_2' = \frac{6}{5}\times\frac{24\times 5}{11} = 13\,\mu C \quad \text{(approx)}$$

$$Q_1' = 1\times\frac{24\times 5}{11} = 11\ \mu C \quad \text{(approx)}$$

$V_1' = 11$ V

$$V_3' = \frac{Q_3'}{C_3} = \frac{13}{3} = 4.3 \text{ V} \quad \text{(approx)}$$

TEST Edge Similar questions can be asked in which one has to find potential across each capacitor or potential difference across a branch.

8. (*c*) **Idea** Accuracy is related to how much close is observed value to the real value, *i.e.,* if real value is 10.2 then reading 9.1 will be more accurate than 7.1. Whereas precision is related to least count of instruments, *i.e.,* 9.112 is more precise than 9.1.

The observation of first person is measured upto two points of decimal but it is far from the actual value. So, it is less accurate but more precise.

TEST Edge Question related to error analysis is important. These are topics and 1 to 2 questions are asked almost every year.

9. (*d*) **Idea** The magnetic field due to a long infinite wire at a perpendicular distance *d* is given by $|\mathbf{B}| = \frac{\mu_0 i}{2\pi d}$ and its direction is given by Right hand thumb rule.

Magnetic field due to wire (I_1) is pointed in downwards direction (according to Right hand thumb rule)

$$\mathbf{B}_1 = \frac{\mu_0 I_1}{2\pi (d\sin\theta)}\cdot(-\hat{\mathbf{k}})$$

[$d \sin\theta$ gives perpendicular distance from wire first.]

Field due to wire I_2 will be pointed in upwards direction.

So, $\mathbf{B}_2 = \frac{\mu_0 I_2}{2\pi (d\sin\theta)}\cdot(\hat{\mathbf{k}})$

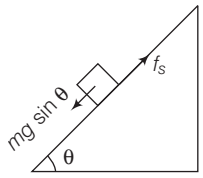
Hence, net field is $\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2$

$$\left[\mathbf{B} = \left[\frac{-\mu_0 I_1}{2\pi(d\cos\theta)} + \frac{\mu_0 I_2}{2\pi(d\sin\theta)}\right]\hat{\mathbf{k}}\right.$$

$$\left.\mathbf{B} = \frac{\mu_0}{2\pi d}\left[\frac{-I_1}{\cos\theta} + \frac{I_2}{\sin\theta}\right]\hat{\mathbf{k}}\right]$$

TEST Edge Question from the application of Biot-Savart's law is asked every year in the examination so students are advised to understand the application of this law.

10. (*d*) If block is at rest f_s must be equal to $mg \sin\theta$.



Here, f_s is equal to less than $(f_s)_{max}$ will depend on the value of inclination (θ).

11. (*a*) Let $p_1 = 1$ atm, $n = 5$ mol, $T_1 = 293$ K

$$V_2 = \frac{V_1}{10}$$

Using $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$,

$$\Rightarrow \quad T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$= 293\,(10)^{0.4} = 736 \text{ K}$$

Now,

$$\text{Workdone} = \frac{nR(T_1 - T_2)}{\gamma - 1}$$

$$= \frac{5 \times 8.3 \times (293 - 736)}{0.4} = -46 \text{ kJ}$$

and $\Delta U = \Delta Q - W = 0 - W = 46$ kJ

12. (*a*) KE of proton and α-particle shall be 1 eV and 2 eV respectively.

Now, momentum, $p = \sqrt{2\,m\,\text{KE}}$

$\therefore$ Momentum of proton is $p_P = \sqrt{2\,m_P \times \text{eV}}$

$\therefore$ Momentum of α-particle whose mass $m_\alpha = 4\,m_P$ is

$$p_\alpha = \sqrt{8\,m_P \times (2\,\text{eV})} = \sqrt{16\,m_P\,\text{eV}}$$

$\therefore$ Now, $\lambda = \frac{h}{p}$ [de-Broglie relation]

$$\Rightarrow \quad \frac{\lambda_P}{\lambda_\alpha} = \frac{p_\alpha}{p_P} = \frac{\sqrt{16\,m_P\,\text{eV}}}{\sqrt{2\,m_P\,\text{eV}}} = 2\sqrt{2} : 1$$

13. (*d*) Slope of line $= -\frac{2}{3}$

Equation of line is $(v - 20) = -\frac{2}{3}(S - 0)$

$$\Rightarrow \quad v = 20 - \frac{2}{3}S \qquad \ldots(\text{i})$$

Velocity at $S = 15$ m,

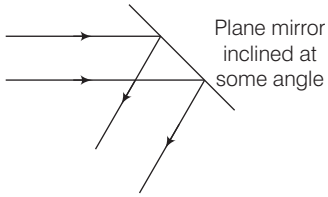
$$i.e., \quad v = \left.\frac{dS}{dt}\right|_{S=15\,\text{m}} = 20 - \frac{2}{3}(15) = 10 \text{ ms}^{-1}$$

Differentiating Eq. (i) w.r.t. time,

$$\text{Acceleration} = \frac{dv}{dt} = -\frac{2}{3}\frac{dS}{dt}$$

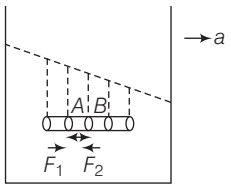
$$\therefore \quad \left.\frac{dv}{dt}\right|_{S=15\text{ m}} = -\frac{2}{3}\left.\frac{dS}{dt}\right|_{S=15\text{ m}} = -\frac{20}{3} \text{ ms}^{-2}$$

14. (*c*) **Idea** If a mirror is rotated at angle θ, then the reflected ray will be rotated through 2θ.



TEST Edge Questions based on size of mirror, distance of image, rotation of mirror are asked in examination. These types of questions are solved with the help of geometry (*e.g.,* congruency of triangles etc.)

15. (*c*) The liquid is accelerated. Let us consider a small part *AB* of a virtual cylinder (horizontal)



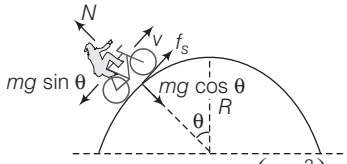
$$F_1 - F_2 = m_{AB} \times a$$

So, $F_1 > F_2$

So, $p_A > p_B$

16. (*d*) **Idea** Whenever a body moves in a circular path, a centripetal force starts to act on the body which is always directed towards the centre.

Here, speed will remains constant due to frictional force.



Now, $N = mg\cos\theta + \left(\frac{mv^2}{R}\right)$

$\downarrow$

constant

As θ decreases $mg\cos\theta$ will increase upto the highest point and then again as θ increase $mg\cos\theta$ will decreases.

So, N will first increase and then decrease.

TEST Edge Question based on uniform circular motion, non-uniform circular motion is asked almost every year some may involve concept of energy conservation.

17. (*b*) **Idea** Whenever there is relative motion between source and observer then frequency of sound is changed and can be calculated with the help of Doppler effect.

Here wall is an observer, so the frequency received by the wall.

$$\Rightarrow \quad f' = \frac{v}{v - v_{source}} f_0$$

$$= \frac{330}{330 - 30} \times 300 = \frac{330}{300} \times 300$$

$$= 330 \text{ Hz}$$

The frequency of sound due to reflection will not change and now wall will act as a source with frequency f' and car will act as an observer.

$$\Rightarrow \quad f'' = \frac{v + v_0}{v} f'$$

$$= \frac{330 + 30}{330} f'$$

$$= \frac{360}{330} \times 330 = 360 \text{ Hz}$$

TEST Edge Questions based on Doppler effect of sound are asked frequently and students are advised to study all possible cases of Doppler effect.

18. (*a*) By law of conservation of angular momentum,

$$\Sigma mvr = (I_{system})\,\omega$$

$$\Rightarrow \quad mv\frac{l}{2} = \frac{(2m)(2l)^2}{12}\omega = \frac{2m(4l^2)}{12}\omega$$

$$\Rightarrow \quad \omega = \frac{3v}{4l} \qquad \text{(anti-clockwise)}$$

Not that clockwise or anti-clockwise rotation can only be determined here by the given figure.

19. (*a*) Diamagnetism is exhibited by all the substances. It arises because of motion of electrons inside an atom. But as effects due to diamagnetism are very weak, So, these can not be observed in paramagnetic and ferromagnetic substances.

20. (*a*) **Idea** When a straight conductor moves with velocity v perpendicular to magnetic field then emf induced is given by $V = Bvl$. Here, it will be $V = Bv(l \sin\theta)$.

Potential difference is $V = Bv(l \sin\theta)$

$$V = 2 \times v \times \frac{30}{100} \times \frac{1}{2} = 0.3\,v$$

Now, $\quad \text{current} = \frac{V}{R}$

$$\Rightarrow \quad 2 = \frac{0.3\,v}{9}$$

$$\Rightarrow \quad v = 60 \text{ m/s}$$

TEST Edge Questions based on induced emf are frequently asked in examination, students are advised to the concept of motional emf and its application as it is very important topic of electromagnetic induction.

21. (*b*) If only conservative forces are acting in the system.

$\Rightarrow W_{ext} = \Delta U$ (if body moves without acceleration)

$\Rightarrow W_{ext} = \Delta U + \Delta KE$ (if body moves with acceleration)

$\Rightarrow \quad W_{conservative} = -\Delta U$ (always)

22. (*a*) Due to insertion of glass plate of refractive index μ of thickness t the optical path becomes μt instead of t. So, net increase in path travelled by wave is, $\mu t - t = (\mu - 1)\,t$.

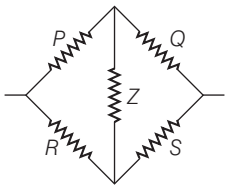
This increase would shift the fringe by a distance of $(\mu - 1)\,t \times \frac{D}{d}$

Now given shift is $(\mu - 1)\,t\,\frac{D}{d} = \frac{2\lambda D}{d}$

$$\Rightarrow \quad (\mu - 1)\,t = 2\lambda$$

$$\Rightarrow \quad t = \frac{2\lambda}{\mu - 1} = \frac{2\lambda}{1.5 - 1} = 4\lambda$$

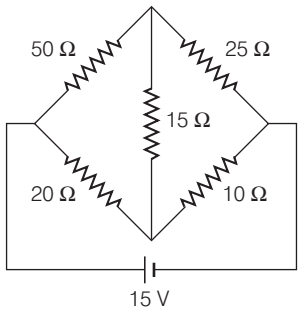
23. (*a*) **Idea** The given problem is application of balanced Wheatstone bridge, *i.e.*,



If $\frac{P}{Q} = \frac{R}{S}$, then Z can be removed.

Given, arrangement is a balanced Wheat-stone bridge

Clearly, $\quad \frac{50}{20} = \frac{25}{10}$



So, 15 Ω resistance can be removed so equivalent resistance is

$$R_{eq} = \frac{75 \times 30}{75 + 30} = \frac{150}{7}$$

So, $\quad \text{current } I = \frac{15}{150} \times 7 = 0.7 \text{ A}$

TEST Edge Concept of Wheat stone bridge can also be applied to capacitor and these problems are in trend.

24. (*b*) From mechanical energy conservation,

$$(TME)_i = (TME)_f$$

$$-\frac{Gm^2}{R} = -\frac{Gm^2}{2r} + 2\left[\frac{1}{2}mv^2\right]$$

$$v = \sqrt{-Gm\left[\frac{1}{R} - \frac{1}{2r}\right]}$$

25. (*a*) $V_{rms}^2 = \dfrac{\int_0^T V^2 dt}{\int_0^T dt}$. Here, $T = 2$ s

Now, $\int_0^T V^2 dt = 2\int_0^{T/2} V\, dt$

[because of symmetry]

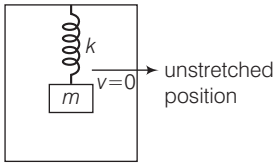
Equation of V from the graph is,

$$V = 2t \qquad [0 \le t \le 1\text{ s}]$$

$$\Rightarrow V_{rms}^2 = \frac{2\times\int_0^1 2t\cdot dt}{2} = 2\int_0^1 t\cdot dt = 2\left[\frac{t^2}{2}\right]_0^1 = 1\text{ V}$$

$$\Rightarrow V_{rms} = 1\text{ Volt}$$

26. (*c*) At the unstretched position, the speed of block will be zero and spring force is also zero.



So, if lift comes in free fall at this moment the box will become at rest with respect to lift and will not execute SHM.

27. (*d*) **Idea** In an isolated system no heat is exchanged between system and surrounding during process.

Here as system is isolated $\Delta Q = 0$, for any part of system in graph.

TEST Edge Question from p-V curve are very important from examination point of view, conceptual clearity is necessary to tackle. These problems, a problem which seem difficult may involve a basic concept with which it can be solved easily. Work done in p-V curve, change in internal energy etc., are common questions which are asked in examination.

28. (*c*) Electromagnetic wave propagate perpendicular to electric and magnetic field vector and in the direction parallel to $\mathbf{E}\times\mathbf{B}$.

29. (*a*) Both Statement I and Statement II are true and Statement II is the correct explanation of the Statement I.

30. (*d*) Positive charge flow from higher potential to lower potential and also potential $\propto 1/r$ for sphere So, smaller sphere have large potential.

Chemistry

31. (*b*) **Idea** This problem is based on concept of n-factor determination and their equivalent wt. determination.

- Determine the n-factor first
- Then, equivalent wt. $= \dfrac{\text{molecular wt.}}{n\text{-factor}}$

Oxidation state of As in $AsO_4^{3-} = x - 2\times 4 = -3$

$$x - 8 = -3$$

$$x = 5$$

Oxidation state of S in $SO_4^{2-} = x - 2\times 4 = -2$

$$x = -2 + 8 = +6$$

n-factor = Sum of positive oxidation states

= Total oxidation state of $2AsO_4^{3-}$ + Total oxidation state of SO_4^{2-}

$= 2\times 5 + 3\times 6 = 10 + 18 = 28$

Equivalent wt. $= \dfrac{M}{28}$

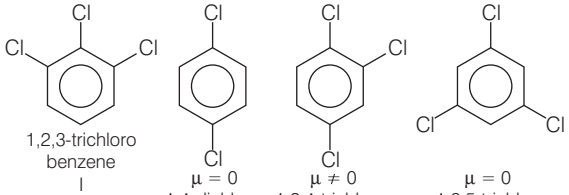
TEST Edge Question related to determination of equivalent wt. of different sols, in different medium may also be asked frequently so, students are advised to study equivalent wt. of $KMnO_4$ in acidic, basic and neutral medium.

32. (*d*) $H_3BO_3 \xrightarrow[-H_2O]{100^\circ C} \underset{(X)}{HBO_2}$ (meta boric acid)

$4HBO_2 \xrightarrow[-H_2O]{160^\circ C} H_2B_4O_7$ (tetra boric acid)

33. (*a*) This problem contains conceptual mixing of structure of compound and its dipole moment.

Structure of given four compounds are as follows



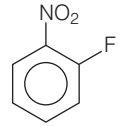
1,2,3-trichloro benzene
I
$\mu \neq 0$

$\mu = 0$
1,4-dichloro benzene
II

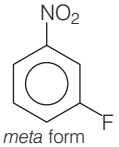
$\mu \neq 0$
1,2,4-trichloro benzene
III

$\mu = 0$
1,3,5-trichloro benzene
IV

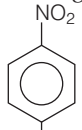
34. (*a*) **Idea** The problem includes conceptual mixing of determination of molecular structure of compound and nucleophilic substitution reaction on substituted benzene ring.



ortho form

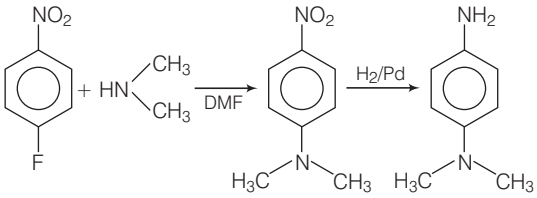


meta form



para form

Seeing at the options given on options, we will choose para form as a starting material and the further sequence of chemical reaction will be as follows.

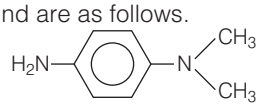


Determination of molecular structure Structure of given compound may be determined as molecular formula = $C_6H_4NO_2F$

Degree of unsaturation (u) $= (C+1) - \frac{H}{2} + \frac{N}{2}$

$$= (6+1) - \frac{5}{2} + \frac{1}{2} = 7 - 2 = 5$$

Since the organic compound is cyclic hence it must contain benzene ring as final product is also aromatic in nature. Hence, the possible structures of compound are as follows.



TEST Edge Students are advised to go through in depth study of various aromatic electrophilic substitution reaction including mechanism of these types of reactions which may also be asked in JEE Main.

35. (*c*) This problem is based on concept of separation of sulphate and sulphite ion. This problem can be solved by using the knowledge of reagent.

$$SO_3^{--}(aq) + SO_4^{--}(aq) \xrightarrow{BaCl_2} \underset{\text{white ppt.}}{BaSO_4} + 2Cl^- + SO_3^{--}$$

Thus, sulphite and sulphate get separated easily.

36. (*c*) **Idea** This problem includes conceptual basis of kinetics of chemical reaction while solving this problem, students are advised to follow the steps

- Write the chemical reaction.
- Write the concentration of each species below it.
- Calculate the value of x_1 and x_2 using information supplied in the question.
- Now, put the values in first order rate equation then come to the correct conclusion.

Let n is the moles of reagent P
when P is reacted with A at time $t = 0$

$$A \longrightarrow B + C$$

$t = 0$	x	0	0
At $t = t$	$n - x$	$2x$	$3x$
At $t = \infty$	0	$2n$	$3n$

$5n = x_2 \Rightarrow n = \frac{x_2}{5}$

$$\{n - x + 2x + 3x\} = x_1$$

$n + 4x = x_1, \quad x = \frac{x_1 - n}{4}$

$$k = \frac{2.303}{t} \log\left(\frac{n}{n-x}\right)$$

Put x and n,

So, $$k = \frac{1}{t} \ln\left(\frac{4\,x_2}{5\,(x_2 - x_1)}\right)$$

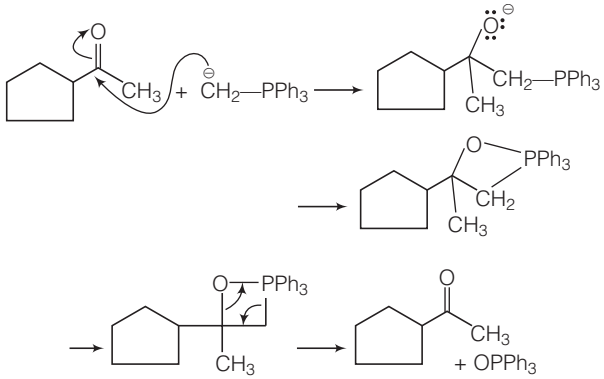
TEST Edge In JEE Main, these types of questions are included to judge the knowledge of student in rate constant involving quantitative approach of student towards solving such type of problem. Question related to rate constant and ideal gas equation can also be asked.

37. (*d*) **Idea** This problem includes conceptual mixing of Witting reaction and Clemenson reduction. Student must have a clear idea about reagent used in various name reaction and rearrangement reaction.

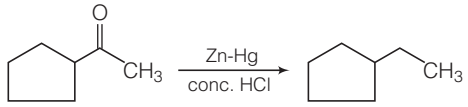
Witting reaction The conversion of cyclopentyl methyl ketone to 2 cyclopentyl 2 methyl ethene by used of Wittig reagent *i.e.*, phosphorous ylid is known as Wittig reaction as shown in *B*.

$\therefore$ [B] = $PPh_3 = CH_2$ or $\overset{\oplus}{P}Ph_3—\overset{\ominus}{C}H_2$

Reaction occurs as follows



Clemenson reduction When carbonyl compound is treated with amalgamated zinc in presence of concentrated HCl, it converts > C═O to > CH_2.



This is a direct method for conversion of ketone to hydrocarbon.

TEST Edge In JEE Main, these types of questions are asked frequently to judge the concept of student in various chemical reaction so student are advised to study the name reactions such as Wolf Kishner reduction, birch reduction, etc., which may also be asked.

38. (*c*) **Idea** While solving this question, students are advised to use the concept of electronic configuration of metal ions and the magnetic moment of ions. Student must follow following tips to solve this problem.

- Write orbital representation of valence shell orbitals using electronic configuration of metal ion.
- Count the number of unpaired electrons and calculate the magnetic moment using

$$\mu = \sqrt{n(n+2)}\ \text{BM}$$

$Ni^{2+} \longrightarrow 3d^8$ | ↿⇂ | ↿⇂ | ↿⇂ | 1 | 1 |

Unpaired electron(s) = 2

Magnetic moment = $\sqrt{n(n+2)} = \sqrt{2(2+2)}$

$= \sqrt{8}$ BM

$Cr^{3+} \rightarrow 3d^3$ | 1 | 1 | 1 | | |

Unpaired electron(s) = 3

Magnetic moment = $\sqrt{3(3+2)} = \sqrt{15}$ BM

$Mn^{2+} \rightarrow 3d^5$ | 1 | 1 | 1 | 1 | 1 |

Unpaired electron(s) = 5

Magnetic moment = $\sqrt{5(5+2)} = \sqrt{35}$ BM

$Fe^{2+} \rightarrow 3d^6$ | ↿⇂ | 1 | 1 | 1 | 1 |

Unpaired electron(s) = 4

Magnetic moment = $\sqrt{4(4+2)} = \sqrt{24}$ BM

TEST Edge This type of question is a general trend which is asked frequently in JEE Main, students are advised to go through study of magnetic moments of coordination compounds also.

39. (*a*) **Idea** This problem includes the concept of Raoult's law and their representation as equation of straight line. While solving this problem, student is advised to follow given tips.

- Write the partial pressure equation using Raoult's law.
- Put the value of partial pressure to calculate mole fractions.
- Rearrange the equation in $\frac{1}{Y_A}$ *vs* $\frac{1}{X_A}$ and determine slope and intercept.

$$p_A = X_A\ p_A^0$$

$$p_B = X_B\ p_B^0$$

and $$Y_A = \frac{p_A}{p_A + p_B} = \frac{p_A^0\ X_A}{p_A^0 X_A + p_B^0 (1 - X_A)}$$

$$Y_A = \frac{p_A^0\ X_A}{X_A\ (p_A^0 - p_B^0) + p_B^0}$$

$$\frac{1}{Y_A} = \left(\frac{p_A^0 - p_B^0}{p_A^0}\right) + \frac{p_B^0}{p_A^0}\ \frac{1}{X_A}$$

So, slope $\frac{p_B^0}{p_A^0}$ and intercept $\frac{p_A^0 - p_B^0}{p_A^0}$.

TEST Edge Students are advised to study Raoult's law for non-ideal solution which may be asked frequently.

40. (*b*) $CaCO_3 \xrightarrow{\text{Heat}} \underset{X}{CaO} + \underset{Y}{CO_2}$

$CaO + C \xrightarrow{\text{Heat}} \underset{C}{CaC_2} + \underset{D}{CO}$

$\underset{C}{CaC_2} + 2H_2O \longrightarrow Ca(OH)_2 + \underset{E}{C_2H_2}$

41. (*a*) **Idea** To solve this problem student is advised to keep in mind the clear understanding of mole concept.

Number of moles = $\frac{\text{Wt. in gram}}{\text{Molecular mass}}$

(I) 0.5 mol of O_3 = 24 g O_3

(II) 3.01×10^{23} molecules of O_2 = 16 g O_2

(III) 5.6 L of $CO_2 = \frac{5.6}{22.4} \times 44 = 11$g CO_2

(IV) 0.5 g atom of O_2 = 8 g

Hence, IV < III < II < I

TEST Edge These types of problems are asked generally in JEE Main to judge the basic mole concept so students are advised to go through clear study of equivalent wt., stoichiometry etc. Question involving this concept may also be asked.

42. (*d*) **Idea** This problem includes conceptual mixig of variom physical properties of haloalkanes *viz*, boiling point, density and polarity.

So, students are advised to go through in depth study of physical properties of hydrocarbons.

Boiling point Boiling point of haloalkane increases with size of attached halogen.

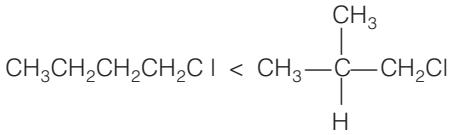
CH_3Cl CH_3Br CH_3I

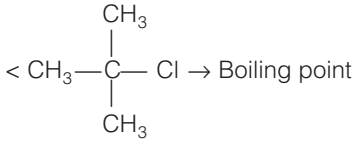
Cl Br I

→ Size of halogen increases

→ Polarity of *C-X* bond increases

Boiling point of haloalkane decreases with increase in branching.





Polarity of C—*X* bond Polarity of C—*X* bond mainly depends upon electronegativilty of halogen atom. But in case of C—Cl and C—F, the bond polarity is explained by size of halogen here C—F bond is less polar than C—Cl due to very small size of F.

$$H_3C—F < CH_3Cl > CH_3Br > CH_3I$$

Density of alkyl halide and water Alkyl chloride and fluoride have lesser density than water while alkyl bromide and alkyl iodide have more density than water.

Hence, correct order of density is

$$CH_3F < CH_3Cl < H_2O < CH_3Br < CH_3I$$

TEST Edge In JEE Main, these questions are asked frequently so students are advised to go through in depth of dipole moment, solubility, stability etc., also, which may also be asked.

43. (*b*) **Idea** This problem can be solved by using the concept of quantitative analysis of inorganic compounds. Look a sight towards option while solving the question and choose the correct possible option using information provided in question.

$$\underset{A}{2FeSO_4} \xrightarrow{\Delta} \underset{D}{Fe_2O_3} + \underset{B}{SO_2} + \underset{C}{SO_3}$$

$$\underset{D}{Fe_2O_3} + 6HCl \longrightarrow \underset{E}{2FeCl_3} + 3H_2O$$

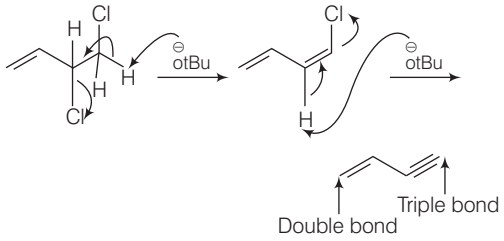
TEST Edge Question based on quantitative analysis of different ions (acid and basic radicals) and dry test of each type of radical may also be asked, so students are recommended to undergo in depth study of wet test and dry test.

44. (*c*) The sols obtained in the two cases will be oppositely charged so coagulate each other.

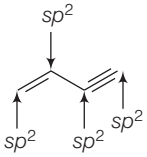
45. (*d*) **Idea** This problem includes conceptual mixing of elimination reaction and type of hybridisation.

While solving this problem, students are advised to complete the reaction and then go through analysis of hybridisation of each carbon atom depending upon number of π-bonds around C-atom.

The above written reaction is an example of elimination reaction occurs as



The hybridisation of carbon atom when it is bonded by single bond, double bond and tripple bond are sp^3, sp^2 and *sp* respectively.



TEST Edge This type of problem including elimination reaction as well as substitution and rearrangement reaction may abo be asked in JEE Main.

The correct order is sp^2, sp^2, *sp* and s*p*.

46. (*d*) **Idea** This problem is based on concept of 'van der Waal's equation' which can be solved by using the value of *b* used in volume correction.

$$b = \frac{4}{3}\pi r^3 + 4N_A$$

and values of compressibility factor at high pressure

i.e., $$Z = 1 + \frac{bp}{RT}$$

van der Waal's equation

$$\left(p + \frac{n^2a}{V^2}\right)(V - nb) = nRT$$

At high pressure, $Z = 1 + \frac{pb}{RT}$

Slope $= \frac{b}{RT}$

$$b = \frac{\pi}{246.3} \times 0.0821 \times 600 \quad \ldots(i)$$

and $$b = \frac{4}{3}\pi r^3 \cdot 4N_A = \frac{16}{3}\pi r^3 N_A \quad \ldots(ii)$$

By Eqs. (i) and (ii), we get

$$\frac{16}{3}\pi r^3 N_A = \frac{\pi}{246.3}\,\text{atm}^{-1} \times 0.0821\,\text{L atm mol}^{-1}\text{K}^{-1} \times 600\,\text{K}$$

$$r^3 = \frac{3 \times 0.0821\,\text{L mol}^{-1} \times 600}{16 N_A}$$

$$r = \sqrt[3]{\frac{3 \times 0.082 \times 600\,\text{L}}{16 \times 0.023 \times 10^{23}}}$$

$$= \sqrt[3]{\frac{3 \times 0.082 \times 600}{16 \times 6.023\, x}} \times 10^{-3}\,m^{-23}$$

$$r = 3\,\text{Å}$$

$$d = 2r = 6\,\text{Å}$$

TEST Edge These types of problems are asked in JEE Main frequently, students are advised to study the concept of van der Waals' equation at low p, high T and low T also, question may also be asked from that area.

47. (*b*) $CaC_2 + H_2O \longrightarrow C_2H_2$

$Be_2C + H_2O \longrightarrow CH_4$

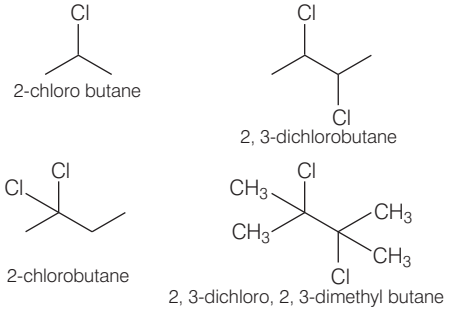
$Mg_4C_3 + H_2O \longrightarrow CH_3—C{\equiv}CH$

All are alkaline earth metals Group (II) elements.

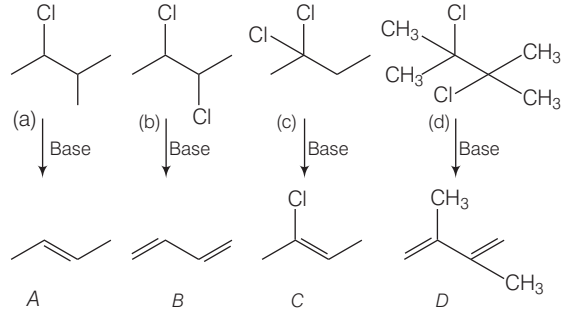
48. (*d*) **Idea** This problem includes conceptual mixing of molecular structure of organic compound, Sytzeff's rule elimination reaction and conjugation.

This problem may be solved by completing the elimination reaction of each reactants and then comparing stability of product using the Sytzeff's rule and conjugation.

1. Molecular structure of given written names of organic compounds are written first as



According to Sytzeff's rule, more substituted (alkylated) alkene are more stable. When the alkyl halide is treated with base, it undergo elimination reaction and produces alkane as follows



Conjugation Greater the conjugation greater will be the stability of product.

Hence, *D* has maximum stability, the correct choice is (d) which is stabilised by conjugation as well as Sytzeff's rule.

TEST Edge Similar type of problem including stability of compound on the basis of inductive effect, resonance, aromatic stability etc., may also be asked in JEE Main frequently.

49. (*c*) **Idea** While solving this problem, students are advised to use the mole concept and stoichiometry as follows

- Consider arbitrary mass of MgC_2O_4 and CaC_2O_4 $(0.7 - x)$ and x respectively.
- Write the chemical reaction and calculate weight of both species.
- Finally using information provided in question complete the further calculation.

Let x g = weight of CaC_2O_4

So, wt. of Mg $C_2O_4 = (0.7 - x)$ g

$$CaC_2O_4 \xrightarrow{\Delta} CaCO_3 + CO_2$$

$$Mg\,C_2O_4 \xrightarrow{\Delta} Mg\,CO_3 + CO_2$$

Weight of $CaCO_3$ produced $= \frac{x}{128} \times 100$

Weight of $MgCO_3$ produced $= \frac{0.7 - x}{112} \times 84$

$$\frac{x}{128} \times 100 + \frac{0.7 - x}{112} \times 84 = 0.47$$

$$x = 0.46\ \text{g}$$

Mol. wt. $CaCO_3 = 100$, $MgCO_3 = 84$, $CaC_2O_4 = 128$, $MgC_2O_4 = 112$

Due to further heating

$$CaCO_3 \xrightarrow{\Delta} \underset{\frac{x}{128}}{CaO} + CO_2$$

$$MgCO_3 \xrightarrow{\Delta} \underset{\frac{7-x}{112}}{MgO} + CO_2$$

Weight of CaO and MgO

$$= \frac{0.46}{128} \times 56 + \frac{0.24}{112} \times 40$$

$$= 0.20 + 0.0857$$

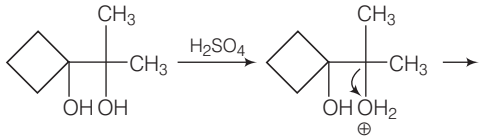
$$= 0.28\ \text{g}$$

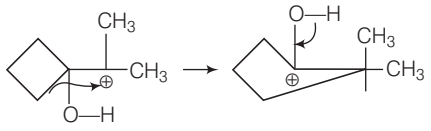
TEST Edge Students are advised to also go through clear idea about calculation of amount of product obtained in various chemical reaction which may be titration, precipitation, combustion reaction or any redox reaction using mole concept and stoichiometry.

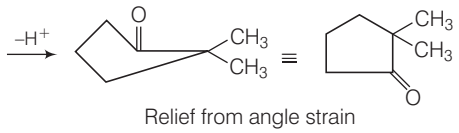
50. (*d*) **Idea** This problem includes conceptual mixing of pinacol pinacolone rearrangement involving rearrangement of carbocation. Follow the given steps sequentialy by go through formation of carbocation and then notice the possibility of rearrangement and finally complete the reaction.

Pinacol-pinacolone rearrangement

The diol is converted into α-hydroxy ketone when reacted in presence of acid is believe to proceeds through rearrangement of carbocation as shown.







Relief from angle strain

Basic of carbocationic rearrangement is due to relief from angle strain.

TEST Edge These types of questions including concept of a carbocationic rearragement are asked frequently. Students are advised to study the Baeyer villiger oxidation, Hoffmann bromamide reaction etc., which may be asked in JEE Main.

51. (*d*) $As_2S_5 + HNO_3 \longrightarrow H_3AsO_4 + NO_2 + H_2O + S$

$H_3AsO_4 + (NH_4)_2\,As\,MoO_4 \longrightarrow$

$(NH_4)_3AsO_4 \cdot 12\,MoO_3$

Yellow precipitate

52. (*d*) $\Delta E = \frac{hc}{\lambda} = \frac{(6.62 \times 10^{-34})(3 \times 10^{8})}{6.11 \times 10^{-8}}$

$= 3.26 \times 10^{-18}$ J

$\Delta E_H = \frac{3}{4}(2.76 \times 10^{-18})$

$= 1.62 \times 10^{-18}$ J

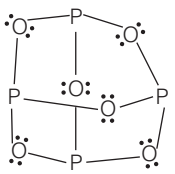
$\Delta E = \Delta E_H (Z^2)$

$Z^2 = \frac{\Delta E}{\Delta E_H} = \frac{3.26 \times 10^{-18}}{1.62 \times 10^{-18}}$

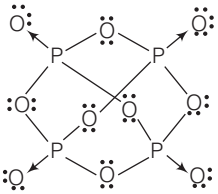
$Z^2 = 2$

$Z = \sqrt{2} = 1.4$

53. (*c*) **Idea** While solving this problem, students are advised to go through structure of oxides of phosphorus, draw the structure of oxide of phosphorus and choose the correct option regarding P_4O_6 and P_4O_{10}.



Structure of P_4O_6

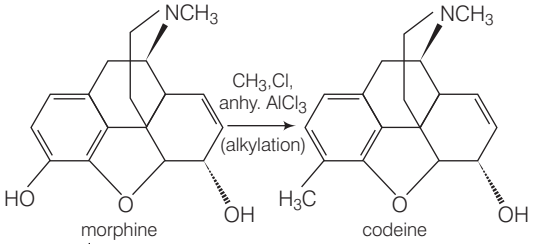


Structure of P_4O_{10}

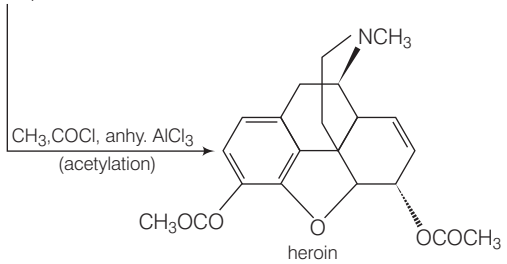
TEST Edge Questions related to structures of oxy acids of sulphur and nitrogen are generally asked, students are advised to go through it.

54. (*c*) **Idea** This problem is directly solved by knowing the Friedal Craft alkylation and Friedal Craft acylation of morphine.

This problem involves conceptual mixing of alkylation and acetylation of morphine. Acetylation and alkylation of morphine gives two different product as follows



morphine codeine



heroin

TEST Edge In general JEE Main similar type of question relating synthesis and characteristics of aspirin and paracetamol may also be asked.

55. (*d*) Equivalents of CO_2 produced $= \frac{I \times n \times t}{96500}$

$= \frac{0.5 \times 0.9 \times 96.5 \times 60}{96500}$

$= \frac{2605.5}{96500} = 0.027$

Moles of CO_2 (n =1) produced ≈ 0.027

Moles of C_2H_6 (n = 2) produced $= \frac{0.027}{2} = 0.0135$

Total moles of gases produced = 0.0405

$$V_{gas} = \frac{nRT}{p}$$

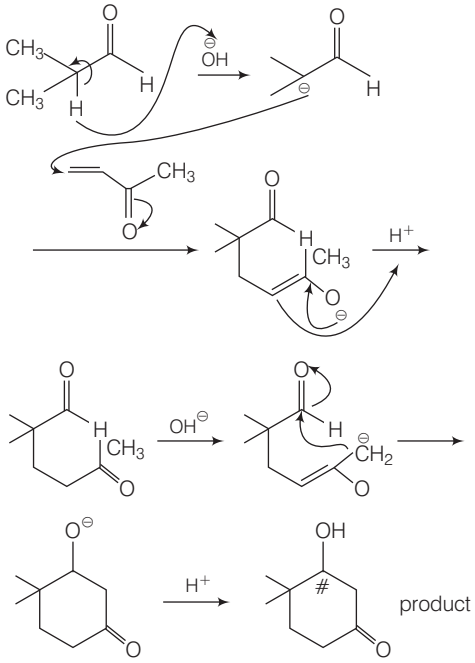
$$= \frac{0.040 \times 0.0821 \times 300}{\frac{760}{760}}$$

$$= 0.997 \text{ L} \approx 1 \text{ L}$$

56. (*a*) **Idea** This problem includes conceptual mixing of Michael addition and number of chiral centre.

- Complete the reaction using the concept of Michael addition.
- Then, complete intramolecular aldol condensation.
- Now, count number of chiral centre.

Michael addition Addition of nucleophile to enone system is done in such a way that the addition looks like addition at 1st and 4th position of enone is known as Michael addition.



The number of chiral centre in product is 1 represented by #.

TEST Edge Problems involving two consecutive reactions are asked frequently in JEE Main. So, students are advised to go step by step completion of reaction, Robinson annelation, benzoin condensation, Cannizzaro reactions are also asked in sequence with some other nucleophilic addition reaction.

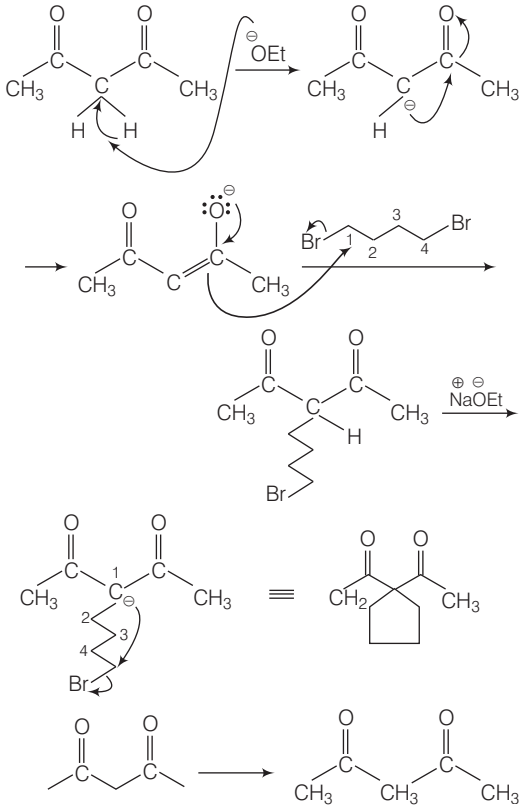
57. (*a*) $Zn(OH)_2$ (*s*) alongwith H^+ shows acidic nature while $Zn(OH)_2$ (*s*) alongwith OH^- shows basic nature. This is why $Zn(OH)_2$ is amphoteric.

58. (*a*) This problem involves conceptual mixing of structure determination of carbonyl compound by using different test and nucleophilic substitution reaction.

Structure determination of acyclic compound having molecular formula = $C_5H_8O_2$

$$u = (5+1) - \frac{8}{2} = 2$$

It does not decolorise the bromine water solution means compound has no C═C bond this confirms presence of two keto groups. Since, it also contain active methylene group mean two keto group must be flanked with —CH_2— group. The compound is 1, 3-diketone and the structure is



Now when this compound is treated with sodium ethoxide produces enolate anion.

The correct answer is choice (a).

TEST Edge Problem occuring *via* carbanionic mechanism are very frequently asked so students are advised to learn the basic principles related to formation of carbanion and their nucleophilic addition reaction.

59. (*c*) This problem includes conceptual mixing of preparation of biodegradable polymer and cause of ecofriendly nature.

Preparation of biodegradable polymer

(i) **Preparation of PHBV** Copolymerisation of 3 hydroxy butanoic acid and 3 hydroxy pentanoic acid produce

poly β-hydroxy butyrate – Co β-hydroxy valerate (PHBV).

(ii) **Preparation of Nylon-2, Nylon 6** Copolymerisation of glycerine and amino caproic acid produces nylon 2, 6.

Cause of Ecofriendly Nature Above prepared both the polymer are ecofriendly due to biodegradable nature of polymer which means it can be reduced to or dissociated to small segments by enzyme or microorganism. The nature of polymer keeps us away from environmental pollution.

Hence, the correct reason is due to easy dissociation of biodegradable polymer to small segments.

60. (*a*) Both Statement I and Statement II are the correct and Statement II is the correct explanation of the Statement I.

Due to greater size of Cl^-, it cannot accomodate in vacant *d* orbital of Si. hence existance of $SiCl_6^{2-}$ do not occurs.

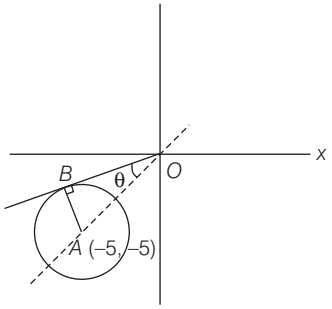
Mathematics

61. (*a*) It is given that,

$$|2z + 10 + 10i| \le 5\sqrt{3} - 5$$

$$\therefore \quad |z + (5 + 5i)| \le \frac{5\sqrt{3} - 5}{2}$$

This equation represents a circle whose centre is $(-5, -5)$ and radius is $\frac{5}{2}(\sqrt{3} - 1)$



Here, $AB = \text{radius} = \frac{5}{2}(\sqrt{3} - 1)$

$$OA = \sqrt{5^2 + 5^2} = 5\sqrt{2}$$

$$\sin\theta = \frac{\frac{5}{2}(\sqrt{3} - 1)}{5\sqrt{2}} = \frac{(\sqrt{3} - 1)}{2\sqrt{2}}$$

$$\therefore \quad \theta = 15°$$

$$\angle XOB = -(135 + 15) = -150 = \frac{-5\pi}{6}$$

62. (*a*) **Idea** Here use the concept of greatest integer function such as $[2 \cdot 1] = 2,\ [3 \cdot 4] = 3$

We have, $2 \cdot 4 \cdot 6 \cdot 8 \cdot 10 \cdot 12 \ldots 1000$

$$= (2 \times 1)(2 \times 2)(2 \times 3)(2 \times 4) \ldots (2 \times 500)$$

$$= 2^{500}\,(1 \times 2 \times 3 \times 4 \times \ldots \times 500)$$

$$= 2^{500}\ 500! \qquad \text{where } [\cdot] \text{ represents GIF}$$

Now, exponent of 2 in 500! is

$$= \left[\frac{500}{2}\right] + \left[\frac{500}{2^2}\right] + \left[\frac{500}{2^3}\right] + \left[\frac{500}{2^4}\right] + \left[\frac{500}{2^5}\right] + \left[\frac{500}{2^6}\right] + \left[\frac{500}{2^7}\right] + \left[\frac{500}{2^8}\right]$$

$$= 250 + 125 + 62 + 31 + 15 + 7 + 3 + 1$$

$$= 494$$

Total exponent of 2 are = 500 + 494 = 994

TEST Edge Generally in JEE Main, number of exponents of natural number in given expansion related questions are asked. To solve these types of questions, students are advised to understand the basic concept of GIF and also acquainted yourself with the concept of factorial.

63. (*c*) **Idea** Here $\int_a^b f(x) = dx = 0$ if $f(x)$ is an odd function such $f(-x) = -f(x)$.

Let us consider

$$H(x) = \frac{f\left(\frac{x^2}{4}\right)[f(x) - f(-x)]}{g\left(\frac{x^2}{4}\right)[g(x) + g(-x)]}$$

$$H(-x) = \frac{f\left(\frac{x^2}{4}\right)[f(-x) - f(x)]}{g\left(\frac{x^2}{4}\right)[g(-x) + g(x)]}$$

$$= \frac{-f\left(\frac{x^2}{4}\right)[f(x) - f(-x)]}{g\left(\frac{x^2}{4}\right)[g(-x) + g(x)]}$$

$$H(-x) = -H(x)$$

$\therefore$ So, it is an odd function

$$\int_{\ln\lambda}^{-\ln\lambda} \frac{f\left(\frac{x^2}{4}\right)[f(x) - f(-x)]}{g\left(\frac{x^2}{4}\right)[g(x) + g(-x)]}\,dx = 0$$

$$\because \quad \int_{-a}^{a} f(x)\,dx = 0$$

If $f(x)$ is odd function.

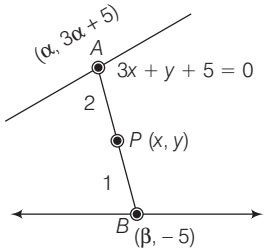
TEST Edge In JEE Main, properties of definite integral based questions are asked. To solve these types of questions, students are advised to learn the properties of definite integral and understand the concept of continuous function.

64. (*d*) Equation of line is

$$3x - y + 5 = 0 \quad \ldots(i)$$

and equation of other line is

$$y = -5 \quad \ldots(ii)$$



Let the general point on the line (i) is $(\alpha, 3\alpha + 5)$ and on line (ii) is $(\beta, -5)$

Using section formula

$$x = \frac{\alpha + 2\beta}{3} \Rightarrow 3x = \alpha + 2\beta \quad \ldots(iii)$$

$$y = \frac{-10 + 3\alpha + 5}{3} \Rightarrow 3y = 3\alpha - 5 \quad \ldots(iv)$$

From Eqs. (iii) and (iv), we get

Solving α and β in terms of x and y

$$\alpha = \frac{3y + 5}{3}, \beta = \frac{9x - 3y - 5}{6}$$

$$l^2 = AB^2 = (\alpha - \beta)^2 + (3\alpha + 10)^2$$

$$l^2 = \frac{1}{4}(3x - 3y - 5)^2 + (3y + 15)^2$$

65. (*a*) Here, A is a 3×3 matrix and matrices B, C and D are related such that $B = \text{adj}(A)$, $C = \text{adj}(\text{adj}(A))$,

$D = \text{adj}(\text{adj}(\text{adj}(A)))$

Let $ABCD = E$, then

$$|\text{adj}(\text{adj adj}(\text{adj}\, E)))| = |E|^{(3-1)^4} = |E|^{16}$$

$$|ABCD|^{16} = |A|^{16}|B|^{16}|C|^{16}|D|^{16}$$

$$= |A|^{16}(|A|^2)^{16}(|A|^4)^{16}(|A|^8)^{16}$$

$$= |A|^{16 + 32 + 64 + 128}$$

$$= |A|^{240}$$

$$\Rightarrow |A|^K = |A|^{240}$$

$$\Rightarrow K = 240 < 256$$

66. (*c*) **Idea** Find the intersection points of the given lines and then get distance from the given point.

Consider the given lines are

$$\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7} = \lambda_1 \quad \ldots(i)$$

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = \lambda_2 \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$4 + \lambda_1 = 1 + 2\lambda_2$$

$$\Rightarrow \lambda_1 - 2\lambda_2 = -3 \quad \ldots(iii)$$

$$-3 - 4\lambda_1 = -1 - 3\lambda_2$$

$$\Rightarrow 4\lambda_1 - 3\lambda_2 = -2 \quad \ldots(iv)$$

From Eqs. (iii) and (iv), we get

$$\lambda_1 = 1, \lambda_2 = 2$$

$\Rightarrow$ Point of intersection $(4 + 1, -3 - 4, -1 + 7)$

i.e., $(5, -7, 6)$

Distance of $(5, -7, 6)$ from $(1, -4, 7)$

$$= \sqrt{16 + 9 + 1} = \sqrt{26}$$

TEST Edge Distance between two lines, intersection of lines related question are asked. To solve these types of questions, students are advised to learn the formula and understand the concept of line.

67. (*c*) We have given inequality as

$$x^2 + 9 < (x + 3)^2$$

$$x^2 + 9 < x^2 + 9 + 6x$$

$$6x > 0 \Rightarrow x > 0 \quad \ldots(i)$$

Again,

$$(x + 3)^2 < 8x + 25$$

$$\Rightarrow x^2 + 9 + 6x < 8x + 25$$

$$\Rightarrow x^2 - 2x - 16 < 0$$

$$\Rightarrow (x - 1)^2 - 17 < 0$$

$$\Rightarrow (x - 1)^2 < 17$$

$$\Rightarrow x \in (1 - \sqrt{17}, 1 + \sqrt{17})$$

Hence, integral value of x are

1, 2, 3, 4, 5

68. (*c*) **Idea** Here $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ and $|\mathbf{a} \cdot \mathbf{a}| = |\mathbf{a}|^2 = 1$

Find the value of of λ.

It is given that the angle between $\mathbf{a}$ and $\mathbf{c}$ is $\cos^{-1}\frac{1}{4}$

So,

$$\mathbf{a} \cdot \mathbf{c} = |\mathbf{a}||\mathbf{c}|\left(\cos^{-1}\frac{1}{4}\right)$$

$$\Rightarrow \mathbf{a} \cdot \mathbf{c} = \frac{1}{4} \quad \ldots(i)$$

Taking dot product by $\mathbf{a} \cdot \mathbf{b} \cdot \mathbf{c}$, we have

$$\mathbf{a} \cdot \mathbf{b} - 2(\mathbf{a} \cdot \mathbf{c}) = \lambda(\mathbf{a} \cdot \mathbf{a})$$

$$\Rightarrow \mathbf{a} \cdot \mathbf{b} - \frac{1}{2} = \lambda$$

$$\Rightarrow \mathbf{a} \cdot \mathbf{b} = \lambda + \frac{1}{2} \quad \ldots(ii)$$

Similarly, $\mathbf{b} \cdot \mathbf{c} = 8 - \frac{\lambda^2}{2} - \frac{\lambda}{4} \quad \ldots(iii)$

and $\mathbf{b} \cdot \mathbf{c} - 2 = \lambda(\mathbf{a} \cdot \mathbf{c}) \quad \ldots(iv)$

From Eqs. (ii), (iii) and (iv), we get

$$8-\frac{\lambda^2}{2}-\frac{\lambda}{4}-2=\lambda\left(\frac{1}{4}\right)$$

$$\Rightarrow \quad \lambda=3,-4$$

TEST Edge Generally in JEE Main, angle between two vectors and different types of questions based on scalar product are asked. To solve these types of questions, students are advised to understand the basic concept of scalar product of two vectors.

69. (*a*) Apart from the owner there are 24 other cars. On his returns 14 cars are left.

$n(S)$ = Number of ways selecting 14 cars out of 24 cars

$= {}^{24}C_{14}$

$n(E)$ = Selecting 14 cars (leaving two neighbouring positions) out of 22 cars

$= {}^{22}C_{14}$

$$P(E)=\frac{{}^{22}C_{14}}{{}^{24}C_{14}}=\frac{15}{92}$$

70. (*d*) Consider the given expression

$$\lim_{x\to\infty}\left(\frac{p^{\frac{1}{x}}+q^{\frac{1}{x}}+r^{\frac{1}{x}}+s^{\frac{1}{x}}}{4}\right)^{3x} \quad [\text{where, } p,q,r,s>0]$$

Put, $\frac{1}{x}=z$, so that $x\to\infty\Rightarrow z\to 0$

$$=\lim_{z\to 0}\left(\frac{p^z+q^z+r^z+s^z}{4}\right)^{\frac{3}{z}}$$

which is in the form 1^∞.

$$=e^{\lim\limits_{z\to 0}\left(\frac{p^z+q^z+r^z+s^z}{4}-1\right)^{3/z}}$$

$$=e^{\lim\limits_{z\to 0}\left(\frac{p^z-1}{z}+\frac{q^z-1}{z}+\frac{r^z-1}{z}+\frac{s^z-1}{z}\right)\frac{3}{4}}$$

$$=e^{\frac{3}{4}(\log_e p+\log_e q+\log_e r+\log_e s)}$$

$$=e^{\frac{3}{4}\log_e(pqrs)}=(pqrs)^{3/4}$$

71. (*a*) **Idea** Find the equation of normal at $(at^2, 2at)$ and then given points are also satisfy the equation of normal.

Here, equation of normal to the parabola $y^2=4ax$ at the point $(at^2, 2at)$ is

$$y+tx=2at+at^3 \qquad \ldots(i)$$

Eq. (i) cuts the parabola again $(aT^2, 2aT)$

Then, $2aT+taT^2=2at+at^3$

$$\Rightarrow \quad 2a(T-t)=-at(T^2-t^2)$$

$$\Rightarrow \quad 2=-t(T+t) \qquad [t\neq T]$$

$$t^2+tT+2=0$$

Since, t is real $T^2-4\cdot 2\cdot 1\geq 0$

$$T^2\geq 8$$

TEST Edge Equation of tangent at parabola and different condition related questions are asked in JEE Main. Students are advised understand the concept of tangent of parabola.

72. (*b*) **Idea** For maxima or minima $f'(x)=0$ then value $x_1, x_2, x_3, \ldots$ obtained if $f(x)>0$ at x_1 then it is maximum.

Here, we have given a function.

$$f(x)=x^{25}(1-x)^{75},\ x\in[0,1]$$

$$f'(x)=25x^{24}(1-x)^{75}-75x^{25}(1-x)^{74}$$

$$=25x^{24}(1-x)^{74}[1-x-3x]$$

$$=25x^{24}(1-x)^{74}(1-4x)$$

For maximum value of $f(x)$, put $f'(x)=0$

$$25x^{24}(1-x)^{74}(1-4x)=0$$

$$x=0,1,\frac{1}{4}$$

also at $x=0$, $y=0$

at $x=1$, $y=0$

and $\quad x=\frac{1}{4}$, $y>0$

So, $f(x)$ attains maximum value at $x=\frac{1}{4}$

TEST Edge The minimum value of function or maximum at open interval or closed interval related questions are asked in JEE Main. To solve these types of questions, students are advised to understand the basic concept of maxima or minima.

73. (*b*) Given that, $e^{2y}+2bxe^y+b^2=0$

Now, simplify the given equation, we get

$$e^{2y}+2bxe^y+b^2x^2=b^2x^2-b^2$$

$$(e^y+bx)^2=b^2(x^2-1)$$

$$e^y+bx=\pm b\sqrt{x^2-1}$$

$$e^y=-b(x\pm\sqrt{x^2-1}) \qquad \ldots(i)$$

Differentiating of Eq. (i) w.r.t. x, we get

$$e^y\frac{dy}{dx}=\frac{-b(x\mp\sqrt{x^2-1})}{\sqrt{x^2-1}} \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\sqrt{x^2-1}\frac{dy}{dx}=\pm 1$$

$$(x^2-1)\left(\frac{dy}{dx}\right)^2=1$$

74. (*c*) **Idea** Here, $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$,

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

and for a quadratic equation

$$ax^2 + bx + c = 0$$

have roots α and β, then

$$\alpha + \beta = -\frac{b}{a},\ \alpha\beta = \frac{c}{a}$$

It is given that,

$$a\cos 2\theta + b\sin 2\theta = c$$

$$\Rightarrow \quad a\left[\frac{1-\tan^2\theta}{1+\tan^2\theta}\right] + b\left[\frac{2\tan\theta}{1+\tan^2\theta}\right] = c$$

$$\Rightarrow \quad a(1-x^2) + 2bx = c(1+x^2) \qquad (\text{let } x = \tan\theta)$$

$$\Rightarrow \quad a - ax^2 + 2bx = c + cx^2$$

$$\Rightarrow \quad (a+c)x^2 - 2bx + (c-a) = 0$$

$$\therefore \quad \text{Sum of roots} = \frac{2b}{a+c}$$

TEST Edge Generally in JEE Main, properties of trigonometric ratio and identities related questions are asked. To solve these types of questions, students are advised to understand the concept of trigonometric ratio.

75. (*b*) We have given that

$$f(x) = [4^x - 2^{x+1} + 1]$$

$$= [2^{2x} - 2^x \cdot 2 + 1]$$

$$f(x) = [(2^x - 1)^2]$$

$$(2^x - 1)^2 \geq 0\ \forall\ x$$

$$\therefore \quad [(2^{x-1})^2]$$

will give all the integral values which is greater than equal to zero, hence all whole numbers are part of solution.

76. (*a*) **Idea** $\because$ $\text{Mean} = \frac{\text{Sum of observations}}{\text{Total number of observations}}$

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2$$

Let the remaining two observations x and y. Since mean of 7 observations is 8.

$$\therefore \frac{2+4+10+12+14+x+y}{7} = 8$$

$$\Rightarrow \quad 42 + x + y = 56$$

$$\Rightarrow \quad x + y = 14$$

also variance $(\sigma^2) = 16$

Since, $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2$

$$\Rightarrow \quad \frac{1}{7}(2^2 + 4^2 + 10^2 + 12^2 + 14^2 + x^2 + y^2) - (8)^2 = 16$$

$$\Rightarrow \quad (4 + 16 + 100 + 144 + 196 + x^2 + y^2) = 80 \times 7 \qquad [\because (x+y)^2 = 14^2 = 196]$$

$$\Rightarrow \quad x^2 + y^2 = 560 - 460 = 100$$

but $\quad (x+y)^2 + (x-y)^2 = 2(x^2 + y^2)$

$$\Rightarrow \quad 196 + (x-y)^2 = 2 \times 100 \qquad [\because (x+y)^2 = 14^2 = 196]$$

$$\Rightarrow \quad (x-y)^2 = 4$$

$$\Rightarrow \quad (x-y) = \pm 2$$

If $(x-y) = +2$, then

$$(x+y) = 14 \text{ and } (x-y) = 2$$

$$\therefore \quad x = 8,\ y = 6$$

If $\quad (x-y) = -2$,

Then, $\quad (x+y) = 14$ and $(x-y) = -2$

$$\Rightarrow \quad x = 6,\ y = 8$$

So, the remaining two observations are 6 and 8.

TEST Edge Median, mode and standard deviation related questions are asked. To solve these types of questions, understand the relation between above.

77. (*d*) The given function is $[x]^2 - [x^2]$

$[x]$ is continuous for all non-integers.

let $\quad x = n \in z$

$$f(x) = n^2 - n^2 = 0$$

Now, if $h > 0$ is small,

$$f(n+h) = [n+h]^2 - [(n+h)^2]$$

$$= n^2 - [n^2 + 2nh + h^2]$$

$$= n^2 - n^2$$

$$= 0 \quad \ldots(\text{i})$$

$$f(n-h) = [n-h]^2 - [(n-h)^2]$$

$$= (n-1)^2 - [n^2 - 2nh + h^2]$$

$$= (n-1)^2 - (n^2 - 1)$$

$$= 2(1-n) \qquad \ldots(\text{ii})$$

From Eqs. (i) and (ii), we have

$$-n + 1 = 0$$

$$\Rightarrow \quad n = 1$$

$\Rightarrow f(x)$ is continuous at $x = 1$.

78. (*c*) Let p : It is raining.

and q : Sky is not filled with clouds.

As we know that
Contrapositive of $p \to q$ is $\sim q \to \sim p$
and converse of $p \to q$ is $q \to p$,
also inverse of $p \to q$ is $\sim p \to \sim q$
So, only option (c) is true.

79. (*d*) It is given that $\sum_{k=1}^{1} D_k = 56$

where, $D_k = \begin{vmatrix} 1 & n & n \\ 2k & n^2+n-1 & n^2+n \\ 2k-1 & n^2 & n^2+n+1 \end{vmatrix}$

$$\Rightarrow \begin{vmatrix} \sum_{n=1}^{n} 1 & n & n \\ \sum_{k=1}^{n} 2k & n^2+n+1 & n^2+n \\ \sum_{k=1}^{n}(2k-1) & n^2 & n^2+n+1 \end{vmatrix} = 56$$

$$\Rightarrow \begin{vmatrix} n & n & n \\ n(n+1) & n^2+n+1 & n^2+n \\ n^2 & n & n^2+n+1 \end{vmatrix} = 56$$

$$\Rightarrow \quad n(n+1) = 56$$

$$\Rightarrow \quad n = 7$$

80. (*b*) **Idea** Here, equation of planes passing through the line of intersection of planes. *i.e.*, $f(x) < f(x_2)$

$\mathbf{r} \cdot \mathbf{n}_1 = d_1$ and $\mathbf{r} \cdot \mathbf{n}_2 = d_2$

is $(\mathbf{r} \cdot \mathbf{n}_1 - d_1) + k(\mathbf{r} \cdot \mathbf{n}_2 - d_2) = 0$

or $\mathbf{r} \cdot (\mathbf{n}_1 + k\mathbf{n}_2) = d_1 + kd_2$, k being any scalar.

It is clear from question that normal vector to planes I and II are $2\hat{\mathbf{i}} - 2\hat{\mathbf{j}}, -\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and vector parallel to line of intersection of the planes.

$$= (2\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) \times (-\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$$

$$= 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$$

$$\text{Required angle} = \cos^{-1}\left(\frac{(4\hat{\mathbf{i}} - \hat{\mathbf{j}}) \cdot (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}})}{\sqrt{17}\sqrt{8}}\right)$$

$$= \cos^{-1}\left(\frac{3}{\sqrt{34}}\right)$$

TEST Edge Equation of a plane parallel to given plane, angle between two planes related questions are asked. To solve these types of questions, understand the basic concept of plane.

81. (*a*) **Idea** $\because$ $f(x)$ is said to be decreasing in D_1 (domain) if $\forall x_1, x_2 \in D$, $x_1 > x_2$. *i.e.*, $f(x_1) < f(x_2)$

Here, $g(x) = f(\sin x) + f(\cos x)$.

Then, differentiate w.r.t. x_1, we get

$$g'(x) = f'(\sin x) \cdot \cos x - f'(\cos x) \cdot \sin x$$

Again differentiate w.r.t. x, we get

$$\Rightarrow \quad g''(x) = -f'(\sin x)\sin x + \cos^2 x\, f''(\sin x) + f''(\cos x)\sin^2 x - f'(\cos x)\cos x > 0$$

$$\forall\, x \in \left(0, \frac{\pi}{2}\right)$$

$\therefore$ $g'(x)$ is increasing in $\left(0, \frac{\pi}{2}\right)$.

Also $g'(\pi/4) = 0$

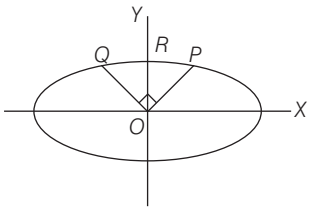
$\Rightarrow$ $g'(x) > 0\ \forall\ x \in (\pi/4, \pi/2)$

and $g'(x) < 0\ \forall\ x \in (0, \pi/4)$

Thus, $g(x)$ is decreasing in $(0, \pi/4)$.

TEST Edge Generally in JEE Main the function is increasing on a given interval, related questions are asked. To solve these types of questions, students are advised to understand the basic concept of monotonicity.

82. (*a*) Consider $x\cos\alpha + y\sin\alpha = p$ be the chord PQ, then p is the desired distance. Homogenizing the equation of the ellipse with the help of this equation, we get the combined equation of OP and OQ.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \left(\frac{x\cos\alpha + y\sin\alpha}{p}\right)^2$$

$$\Rightarrow \quad \left(\frac{1}{a^2} - \frac{\cos^2\alpha}{p^2}\right)x^2 + \left(\frac{1}{b^2} - \frac{\sin^2\alpha}{p^2}\right) y^2 - \frac{2xy\sin\alpha\cos\alpha}{p^2} = 0$$

As, $OP \perp OQ$

$$\therefore \quad \frac{1}{a^2} - \frac{\cos^2\alpha}{p^2} + \frac{1}{b^2} - \frac{\sin^2\alpha}{p^2} = 0$$

$$\Rightarrow \quad p = \frac{ab}{\sqrt{a^2+b^2}}$$

83. (*a*) **Idea** Draw the graph of $y = x^3$ and its normal at (1, 1)

$\because$ Equation of normal at (x_1, y_1) is

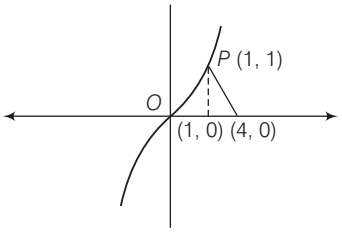
$$y - y_1 = -\frac{dx}{dy}(x - x_1)$$

Equation of the curve is given as

$$y = x^3$$

$$\frac{dy}{dx} = 3x^2$$

$$\left(\frac{dy}{dx}\right)_{(1,1)} = 3$$



Normal at $P(1,1)$ is

$$(y-1) = -\frac{1}{3}(x-1)$$

$$3y + x = 4$$

$$\frac{x}{4} + \frac{y}{\frac{4}{3}} = 1$$

So, normal intersect x-axis at (4, 0).

Hence, required area $= \int_0^1 x^3 dx$ + area of triangle

$$= \left[\frac{x^4}{4}\right]_0^1 + \frac{1}{2} \times 3 \times 1$$

$$= \frac{1}{4} + \frac{3}{2} = \frac{7}{4}$$

TEST Edge Area bounded by the different types of curve related questions are asked. To solve these types of questions, students are advised to understand the concept of graph transformation of different curves.

84. (*b*) Here the relation R of X is defined by

$$R, \{(A, B) \mid A \cap B = \phi\}$$

Now, since $A \cap A \neq \phi$

$\Rightarrow$ $A\ R\ A$

$\Rightarrow$ R is not reflexive,

Let $A, B \in R$

such that $A\ R\ B$

$\Rightarrow$ $A \cap B = \phi$

$\Rightarrow$ $B \cap A = \phi$

$\Rightarrow$ $B\ R\ A$

$\Rightarrow$ R is symmetric.

R is not transitive.

For example, take

$$A = \{1, 3\}., B = \{2, 4\}, C = \{3, 6\}$$

Then, $A \cap B = \phi$ & $B \cap C = \phi$

But $A \cap C = \{3\} \neq \phi$

So, $A\ R\ B$ & $B\ R\ C \Rightarrow ARC$

$\Rightarrow R$ is not transitive.

85. (*b*) **Idea** $\because$ If lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ are coplanar, then

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

If the given lines are coplanar, then

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & x_1 & x_2 \\ 3 & 4 & 5 \end{vmatrix} = 0 \ \because \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$\Rightarrow$ $2x_1 - x_2 = 2$ …(i)

and $x_1t^2 + (x_2 + 2)t + a = 0$

Sum of roots $= -\frac{(x_2+2)}{x_1}$

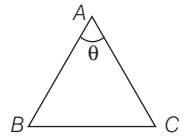
$$= -\frac{(2x_1 - 2 + 2)}{x_1} = -\frac{2x_1}{x_1}$$

Sum of roots $= -2$

TEST Edge Equation of line which lies on plane and equation of line in different form related questions are asked. To solve these types of questions, learn the formula of equation of line in various form.

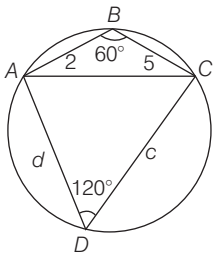
86. (*a*) **Idea** Sum of opposite angles of a cyclic quadrilateral is 180°.

For a triangle



area of ΔABC is $\frac{1}{2} AB \cdot AC \sin\theta$

Here, $ABCD$ is cyclic quadrilateral with



$AB = 2$ and $BC = 5$

$\angle ABC = 60°$ (given)

$\because$ The quadrilateral is cyclic

$\Rightarrow$ $\angle CDA = 180° - 60°$

$= 120°$

Let $CD = c$ and $DA = d$

Area of quadrilateral $ABCD$ = area of ΔABC + area of ΔACD

$$= \frac{1}{2} \cdot AB \cdot BC \sin 60° + \frac{1}{2} CD \cdot DA \times \sin 120°$$

$$= \frac{1}{2} \cdot 2.5 \frac{\sqrt{3}}{2} + \frac{1}{2} cd \frac{\sqrt{3}}{2}$$

Area of quadrilateral $ABCD = \frac{5\sqrt{3}}{2} + \frac{1}{2} cd \frac{\sqrt{3}}{2}$

$$\Rightarrow \quad 4\sqrt{3} = \frac{5\sqrt{3}}{2} + \frac{\sqrt{3}}{4} cd$$

$$\Rightarrow \quad \frac{\sqrt{3}}{4} cd = 4\sqrt{3} - \frac{5\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$$

$$\Rightarrow \quad cd = 6 \qquad \ldots(i)$$

also $AB^2 + BC^2 - 2AB\,BC \cos 60° = AC^2$

$$= CD^2 + DA^2 - 2CD\,DA \cos 120°$$

{by cosine rule}

$$\Rightarrow \quad c^2 + d^2 + cd = 19$$

$$\Rightarrow \quad c^2 + d^2 = 13 \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$c^2 + d^2 = 13$$

and $$c^2 d^2 = 36$$

$\Rightarrow c^2$ and d^2 are roots of the equation

$$t^2 - 13t + 36 = 0$$

$$\Rightarrow \quad t = 9, 4$$

$$\Rightarrow \quad c^2 = 9, d^2 = 4$$

or $$c^2 = 4, d^2 = 9$$

or $$c = 2, d = 3$$

Hence, other two sides are 2 and 3.

TEST Edge In different form based on quadrilateral related questions are asked. To solve these types of questions, understand the basic concept of quadrilateral and learn the formulae of area of these quadrilaterals.

87. (*c*) Consider the given expression

$(abc + abd + acd + bcd)^{10}$

$$= a^{10} b^{10} c^{10} d^{10} \left[\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}\right]^{10}$$

Coefficient of $a^8 b^4 c^9 d^9$ in

$(abc + abd + acd + bcd)^{10}$

= Coefficient of $a^{-2} b^{-6} c^{-1} d^{-1}$

$$\text{in} \left[\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}\right]^{10}$$

$$= \frac{10!}{2!\,6!\,1!\,1!}$$

$$= \frac{10 \times 9 \times 8 \times 7}{2}$$

$$= 2520$$

88. (*b*) **Idea** For an AP such $a_1, a_2, a_3, \ldots$ and common difference be d then $a_2 = a_1 + d$ and follow the conditions given in question.

Let the common difference of the AP be d.

$$a_{2r} = a_{2r-1} + d$$

$$\Rightarrow \quad \Sigma a_{2r} = \sum_{r=1}^{10^{99}} (a_{2r-1} + d)$$

$$\Rightarrow \quad \sum_{r=1}^{10^{99}} a_{2r} = \sum_{r=1}^{10^{99}} a_{2r-1} + 10^{99} d$$

$$\Rightarrow \quad 100^{100} = 10^{99} + 10^{99} d$$

$$\Rightarrow \quad 10 = 1 + d$$

$$\Rightarrow \quad d = 9$$

TEST Edge Sum of n terms of an AP and find the nth or rth term of an AP related questions are asked. To solve these types of questions, students are advised to understand the basic concept of AP.

89. (*b*) I. Centre of the given $C(2, 1)$ and radius is 5.

$\therefore$ Distance of $P(10,7)$ from

$$C(2,1) = \sqrt{(10-2)^2 + (7-1)^2} = \sqrt{64 + 36} = 10$$

$\therefore$ Greatest distance = 5 + 10 = 15

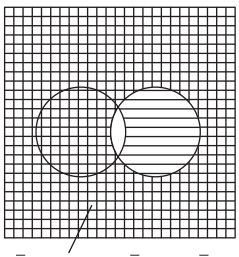
and minimum distance = 10 – 5 = 5

II. It is also true but it is not a correct explanation of the Statement I.

90. (*d*) We know that

$$P\left(\frac{A}{\overline{B}}\right) + P\left(\frac{\overline{A}}{\overline{B}}\right) = \frac{P(A \cap \overline{B})}{P(\overline{B})} + \frac{P(\overline{A} \cap B)}{P(\overline{B})}$$

$$= \frac{P(A \cap \overline{B}) + P(\overline{A} \cap B)}{P(\overline{B})} = \frac{P(\overline{B})}{P(\overline{B})} = 1$$



$P(\overline{B}) = P(A \cap \overline{B}) + P(\overline{A} \cap B)$

Statement II $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

By definition of conditional probability.

$$P(\overline{B}) = P[u \cap \overline{B}] = P(A \cup \overline{A}) \cap \overline{B})$$

$$= P(A \cap \overline{B}) \cup P(\overline{A} \cap \overline{B})$$

$$= P(A \cap \overline{B}) + P(\overline{A} \cap \overline{B})$$

Statement I is false but Statement II is true. So, option (d) is correct.