JEE-MAIN 2015

MODEL GRAND TEST PAPER - I

KEY SHEET

PHYSICS :

1) 2	2) 3	3) 3	4) 2	5) 3	6) 1	7) 4	8) 3	9) 3	10) 3
11) 2	12) 1	13) 4	14) 4	15) 4	16) 2	17) 3	18) 1	19) 1	20) 4
21) 1	22) 1	23) 4	24) 1	25) 3	26) 3	27) 2	28) 2	29) 2	30) 2

MATHEMATICS :

1) 3	2) 1	3) 3	4) 2	5) 3	6) 3	7) 4	8) 4	9) 2	10) 3
11) 2	12) 3	13) 3	14) 3	15) 2	16) 2	17) 3	18) 2	19) 2	20) 4
21) 2	22) 3	23) 3	24) 4	25) 4	26) 2	27) 2	28) 2	29) 2	30) 4

CHEMISTRY :

1) 4	2) 3	3) 2	4) 2	5) 1	6) 1	7) 2	8) 1	9) 3	10) 3
11) 1	12) 4	13) 3	14) 2	15) 4	16) 4	17) 2	18) 1	19) 1	20) 4
21) 4	22) 3	23) 3	24) 1	25) 2	26) 3	27) 4	28) 3	29) 3	30) 4

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MODEL GRAND TEST PAPER - I

<u>HINTS & SOLUTIONS</u>

<u>PHYSICS</u>

1. Nuclear density is of the order of $10^{17} kg / m^3$
2. Use parallel axes theorem

   $I_{rem} = I_{tot} - I_{removed}\ , I_{rem} = \frac{MR^2}{2} - \left[\frac{MR^2}{32} + \frac{MR^2}{16}\right]$

3. $L.C = 1MSD - 1VSD = 1mm - \frac{99}{100}mm$
4. follow color coding Table
5. Momentum is a vector quantity. Photons may move in opposite directions
6. Verify dimensions
7. $T = 2\pi\sqrt{\frac{L}{2g}}, L = \frac{M}{Ad}$
8. $\gamma = 1 + \frac{2}{f}$
9. $\frac{P}{Q} = \frac{1000}{10} = 100$

   The third resistance $R = 999\Omega$

   Let 'X' be the unknown resistance Applying formula

   $\frac{P}{Q} = \frac{R}{X}$

   $X = \frac{P}{Q}(R) = \frac{1}{100}(999) = 9.99\Omega$

10. $E = 12400 / \lambda$
11. $i_d = V_{rms}\,\omega\,C$

    $= 230 \times 600 \times 200 \times 10^{-12} = 27.6\ \mu A$

12. Using $\lambda = \frac{h}{\sqrt{2km}}$ *we find* $\lambda\,\alpha\,\frac{1}{\sqrt{k}}$
13. Using $v_d\ \alpha\ \frac{1}{n}$ *we get* $\frac{\upsilon_{de}}{\upsilon_{dh}} = \frac{l_e}{l_h} \times \frac{n_h}{n_e} = -\frac{7}{4} \times \frac{5}{7} = \frac{5}{4}$

14. 1) $M = \frac{-f_0}{f_e} = \frac{-200}{5} = -40$

2) for distinct vision

$$M = \frac{-f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$$

$$M = -40\left(1 + \frac{5}{25}\right)$$

$$M = -40\left(\frac{6}{5}\right) = -48$$

15. $$\frac{d\theta}{dt} = k\left(\frac{\theta_1 + \theta_2}{2} - \theta_s\right)$$

$$\frac{30}{20} = \frac{2}{10} \times \frac{x}{2}$$

16. Using $S = \frac{I_g}{I - I_g} G, we\ get = \frac{2}{5-2} \times 12 = 8\Omega$

Shunt is used in parallel.

17. From basic knowledge.

18. (1) Here $R = \frac{u^2 \sin 2\theta}{g} = \frac{(10)^2 \sin 60^0}{10} = 5\sqrt{3} = 8.66m.$

19. (1) Here frictional force is equal to the weight of the body.

$F_f = W = mg$

$\mu R = mg = W \qquad W = 0.2 \times 10 = 2N.$

20. (4) Since A is expanded isothermally, then $PV = P_{1.} 2V \quad or \quad P_1 = \frac{P}{2}$

Again since gas is expanded adiabatically.

$\therefore PV^\gamma = P_2 (2V)^\gamma \quad or \quad P_2 = \frac{P}{2^\gamma}$

So, $\frac{P_1}{P_2} = \frac{2^\gamma}{2} = 2^{\gamma - 1}$

21. (1) The frequency of beats for two open organ pipes ae given as $n_1 = \frac{v}{2l}$ and $n_2 = \frac{v}{2(l + \Delta l)}$

Therefore, beats frequency $n = n_1 - n_2$

$$= \frac{v}{2l} - \frac{v}{2(l + \Delta l)} = \frac{v}{2}\left[\frac{l + \Delta l - l}{l(l + \Delta l)}\right] = \frac{v}{2}\frac{\Delta l}{l(l + \Delta l)} = \frac{v\Delta l}{2l^2}$$

22. (1) The diffraction of X-rays can take place only when spacing between two adjacent planes is of the order of wavelength of X-rays. It happens so in case of crystals. So, X-rays are used for structural analysis of crystals.

23. (4) The capacitance of first capacitor $C_1 = 3\mu F$
The capacitance of second capacitor $C_2 = 2\mu F$
Voltage across first capacitor $V_1 = 300V$
Voltage across second capacitor $V_2 = 200V$
In parallel combination, charge across first capacitor $q_1 = C_1V_1 = 3\times 300 = 900\ \mu C$
Charge across second capacitor $q_2 = C_2V_2 = 2\times 200 = 400\ \mu C$
Therefore, total charge in circuit $q = q_1 + q_2 = 900 + 400 = 1300\ \mu C$
and common voltage is 100 V, so the charge flow through the connecting wire is $600\mu C$

24. (1) Susceptibility of ferromagnetic substance is very high and positive

25. (3) The frequency f the electron $= 1.6\times 10^{15}$ / sec

$$\therefore i = \frac{charge\,on\,electron(q)}{time(t)}$$

= charge (q) $\times$ frequency of electorn (v)

$= 6.6\times 10^{-19}\times 6.6\times 10^{15} = 1.06\times 10^{-3}$ amp

Now magnetic filed at the centre is

$$B = \frac{\mu_0 i}{2R} = \frac{4\pi\times 10^{-7}\times 1.06\times 10^{-3}}{2\times 53\times 10^{-12}} = 12.5\ \text{tesla}$$

26. $$Y = \frac{F/A}{\Delta x/\ell} \Rightarrow Y = \frac{F}{\Delta x}\times\frac{\ell}{A} \Rightarrow Y = k\frac{\ell}{A}$$

$(F = k\Delta x)$

27. (2) In steady state

$$\frac{Q}{t} = \frac{K_1A(\theta_1-\theta)}{1} \Rightarrow \frac{Q}{t} = \frac{K_2A(\theta-\theta_2)}{1}$$

θ = temperature of interface

$$K_1(\theta_1-\theta) = K_2(\theta-\theta_2) \qquad 5(100^0-\theta) = 3(\theta-20^0)$$

$$500^0 - 5\theta = 3\theta - 60^0 \quad 8\theta = 560^0 \qquad \text{Hence, } \theta = \frac{560^0}{8} = 70^0C$$

28. (2) At equilibrium just before bursting the force due to surface tension is balanced by electrostatic repulsion so,

$$\frac{4T}{r}\times Area = \frac{\sigma^2}{2\varepsilon_0}\times Area$$

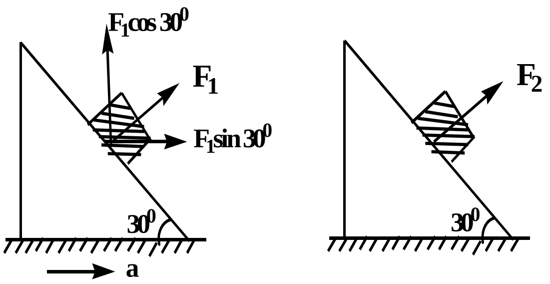
$$\Rightarrow r = \frac{8\varepsilon_0 T}{\sigma^2}$$

29. Surface is smooth. Therefore, force of friction between the block and the plane is zero. So, contact force is really the normal reaction between the two. In the first case

$F_1 \sin 30^0 = ma$ and $F_1 \cos 30^0 = mg$ or $F_1 = \frac{mg}{\cos 30^0}$

And in the second case $F_2 = mg\cos 30^0$

$\therefore \frac{F_1}{F_2} = \frac{1}{\cos^2 30^0} = \frac{4}{3}$



30. $Q \propto AT^4$ and $\lambda_m T =$ constant, Hence,

$Q\alpha \frac{A}{(\lambda_m)^4}$ or $Q\alpha \frac{r^2}{(\lambda_m)^4}$

$Q_A : Q_B : Q_C = \frac{(2)^2}{(3)^4} : \frac{(4)^2}{(4)^4} : \frac{(6)^2}{(5)^4} = \frac{4}{81} : \frac{1}{16} : \frac{36}{625} = 0.05 : 0.0625 : 0.0576$

i.e., Q_B is maximum.

Mathematics

31. Z lies on or inside an ellipse with foci (-3, 0) and (1, 0) and (4, 0) is a point out side the ellipse. Its min and max distances from any point on the ellipse are 1 and 9

32. $S = (n-1)\cos\frac{2\pi}{n} + (n-2)\cos\frac{4\pi}{n} + (n-3)\cos\frac{6\pi}{n} + ... + \cos\frac{2(n-1)\pi}{n}$

$S = 1\cos\frac{2\pi}{n} + 2\cos\frac{4\pi}{n} + ... + (n-1)\cos\frac{2(n-1)\pi}{n}$

$2S = n\left(\cos\frac{2\pi}{n} + \cos\frac{4\pi}{n} + ... + \cos\frac{2(n-1)\pi}{n}\right)$

$$2S = n\frac{\sin(n-1)\frac{\pi}{n}}{\sin\frac{\pi}{n}}\cos\left(\frac{\frac{2\pi}{n} + \frac{2(n-1)\pi}{n}}{2}\right) = -n$$

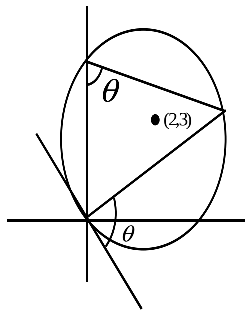
33. $g'(c) = \dfrac{\dfrac{f(5)}{6} - \dfrac{f(0)}{1}}{5}$ (from L.M.V.T)

$= \dfrac{-\dfrac{1}{6} - 4}{5} = \dfrac{-25}{6\times 5} = -\dfrac{5}{6}$

34. Let $x^2 = 4\sin\theta, y^2 = \dfrac{8}{3}\cos\theta \Rightarrow x^2 + y^2 = 4\sin\theta + \dfrac{8}{3}\cos\theta$

$\therefore$ maximum value $= \sqrt{16 + \dfrac{64}{9}} = \dfrac{4\sqrt{13}}{3}$

35



Equation of tangent at origin is

$-2(x+0) - 3(y+0) = 0$

$\Rightarrow 2x + 3y = 0$

$\tan\theta = \dfrac{7}{4}$

$\Rightarrow \left| \dfrac{m + \dfrac{2}{3}}{1 - \dfrac{2m}{3}} \right| = \dfrac{7}{4} \qquad \Rightarrow \dfrac{3m+2}{3-2m} = \dfrac{7}{4}$

$\Rightarrow 12m + 8 = 21 - 14m \qquad \Rightarrow 26m = 13 \Rightarrow m = \dfrac{1}{2}$

$\therefore y = \dfrac{1}{2}x \Rightarrow x - 2y = 0$

36. $f'(x) = (x-2)^{2/3}2 + (2x+1)\dfrac{2}{3}(x-2)^{-1/3}$

$= \dfrac{6(x-2) + 2(2x+1)}{3(x-2)^{1/3}} = \dfrac{10x - 10}{3(x-2)^{1/3}}$

$x = 1$ is a point of maximum and $x = 2$ is a point of minimum

$\therefore$ No.of extremum points is 2

37. $\left(\bar{a}\times\bar{b}\right)\times\bar{b}=\left(\bar{b}.\bar{a}\right)\bar{b}-\left(\bar{b}.\bar{b}\right)\bar{a}$

$=2(2\bar{i}-\bar{j}-\bar{k})-6(\bar{i}+\bar{j}+\bar{k})$

$=-2\bar{i}-8\bar{j}-8\bar{k}$

38. $n(s)=8_{C_3}=56$

Choosing a vertex 3 triangles can be formed from the diagnols of the faces not passing through the vertex. But each triangle is repeated 3 times....$n(E)=\frac{8\times 3}{3}=8$

$\therefore p(E)=\frac{8}{56}=\frac{1}{7}$

39. A is symmetric $\Rightarrow A' = A$ now $X'AY=\left((X'AY)'\right)$

$=(Y'A'X)$

$=(Y'AX) \qquad (\because\ X'AY \text{ is } 1\times 1 \text{ matrix})$

Let $E_1=\begin{bmatrix}1\\0\end{bmatrix},\ E_2=\begin{bmatrix}0\\1\end{bmatrix}$

If $X=E_1\ and\ Y=E_2\ then$

$E^1{}_1AE_2=E^1{}_2\cap E_1\Rightarrow a_{12}=a_{21}$

$\therefore$ A is symmetric.

40. For $(x,y)\in R\Rightarrow x=wy$

but $(y,x)\in R\Rightarrow R\Rightarrow y=xw\Rightarrow x=y$

$\therefore$ R is not symmetrix.

41. $\int\frac{x^2-1}{x^2\left(x+\frac{1}{x}\right)\sqrt{x^2+\frac{1}{x^2}}}dx=\int\frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)\sqrt{\left(x+\frac{1}{x}\right)^2-2}}dx$ put $x+\frac{1}{x}=t$

$$=\int\frac{dt}{t\sqrt{t^2-2}} \qquad t=\sqrt{2}\sec\theta$$

$$=\int\frac{\sqrt{2}\sec\theta\tan\theta d\theta}{\sqrt{2}\sec\theta\sqrt{2}\tan\theta}=\frac{1}{\sqrt{2}}\theta+C$$

42. Put $x^3=t$

$$\therefore I_2=\frac{1}{3}\int_0^1\frac{dt}{e^t(2-t)}=\frac{1}{3}\int_0^1\frac{e^t}{e(1+t)}dt=\frac{1}{3e}I_1$$

$$\Rightarrow\frac{I_1}{I_2}=3e$$

43. Let $x_i - 8 = \times$

$$\sum_{i=1}^{18} \times = 9, \ \sum_{i=1}^{18} \times^2 = 45$$

$$\therefore S.D = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2} \quad = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

44. $\frac{x-1}{1} = \frac{y-2}{-2} = \frac{3-3}{2} = \lambda$

$$\left.\begin{matrix} point\ on\ the \\ plane \end{matrix}\right\} (\lambda+1, -2\lambda+2, 2\lambda+3)$$

$= \lambda + 6 = 11 \quad \lambda = 5$

$(6, -8, 13)$

45. Normals $\Rightarrow \left(-3, \frac{3}{2}\right)$

Centre of given circle =(2,3/2)

Radius =5/2

Radius of required circle = 5 + 5/2

= 15/2

Diameter =15.

46. $y = mx + \sqrt{a^2m^2 - b^2}$

$y - mx - \sqrt{a^2m^2 - b^2} = 0$

$y^2 + m^2x^2 - 2mxy = a^2m^2 - b^2$

$k^2 + m^2b^2 - 2mhk = a^2m^2 - b^2$

$m^2\left(b^2 - a^2\right) - 2mhk + b^2 + k^2 = 0$

$\frac{b^2 + k^2}{h^2 - a^2} = c^2$

$b^2 + y^2 = c^2\left(x^2 - a^2\right)$

47. $\frac{dy}{dx} + 2xy = 2x$

$e^{\int 2x dx} = e^{x^2}$

$ye^{x^2} = \int e^{x^2} 2x dx = e^{x^2}$ + c

$y = 1 + \frac{c}{e^{x^2}}$

$\underset{x \to \infty}{Lt} \quad y = 1$

48. $2y\frac{dy}{dx}=4$, $\frac{dy}{dx}=\frac{2}{y}$, $-\frac{dx}{dy}=\frac{-y}{2}=1$, $y=-2$, $x=1$

Chord $=y+2=(x-1)$ solve with the curve.

49. Successive application of L 'hospital's rule.

50. Function need not be invertible.

51. If $x^2+x=y$ then $y^2-5y-6=0$

$$(y^2-6)(y+1)=0$$

$$x^2+x-6=0,\ x^2+x+1=0$$

$$x=3,-2,\qquad x=\omega,\omega^2$$

Sum of non-real roots = $\omega+\omega^2=-1$

52. Take $f(x)=x\sin x$, which is continuous and differentiable $f(0)=f(\pi)=0$

By Roll'e theorem, there exists at least one root is $f^1(x)=0\Rightarrow x\cos x+\sin x=0$

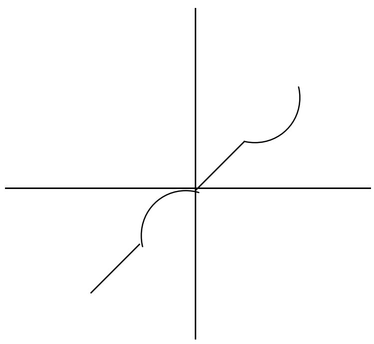
53. no. of orderd pairs = $(2(2)+1)(2(4)+1)(2(2)+1)$

$=5\times9\times5=225$

54. $a_2-a_1=1, a_3-a_2=2, a_4-a_3=3,\ldots$

$$a_n=1+\frac{n(n-1)}{2}=\frac{n^2-n+2}{2}$$

$$\text{sum}=\frac{1}{2}\left\{\frac{30.31.61}{6}-\frac{30.31}{2}+2.30\right\}=4525$$

55. 

From graph

56. $\frac{1}{2r^2}=\frac{2}{4r^2}=\frac{(2r+1)-(2r-1)}{1+(4r^2-1)}$

$$sum=\sum\left(\tan^{-1}(2r+1)-\tan^{-1}(2r-1)\right)$$

$$=\tan^{-1}(2r+1)-\tan^{-1}(1)$$

$$=\tan^{-1}\left(\frac{n}{n+1}\right)$$

57. $p \rightarrow (q \rightarrow p) = \sim p \vee (q \rightarrow p)$

$= \sim p \vee (\sim q \vee p)$

$= \sim p \vee p \vee q \quad = p \rightarrow (p \vee q)$

58. $(1+63)^{x} - (63-1)^{2x+1}$

Reminder is 2.

59. Total Function $= 5^3$

60. Let $\ell = \int_0^x [cot\, x]dx \Rightarrow \ell = \int_0^x [cot(\pi - x)]dx$

$\therefore \ell + \ell = \int_0^x (-1)dx \quad (\because [x]+[-x] = -1 \text{if } x \notin z) = 0 \text{ if } x \in z\ ;\ \ell = -\frac{\pi}{2}$

Chemistry

61. Fluorine being more electronegative than carbon bond pairs mole towards fluorine In C-H bonds bond pairs move towards carbon. Hence all bond angles are not equal. As bond angle decreases (in F-C-F) s- character of hybrid orbitals also decreases . The molecule has obstorted tetrahedral shape.

62. equal to the one third of body diagonal for fcc.

63. Second carbon from nitrogen is only chiral.

64. (2) sp^3d^2 is octahedral 6 bond angle are with 90^0

65. $\Delta x = \sqrt{3}a$

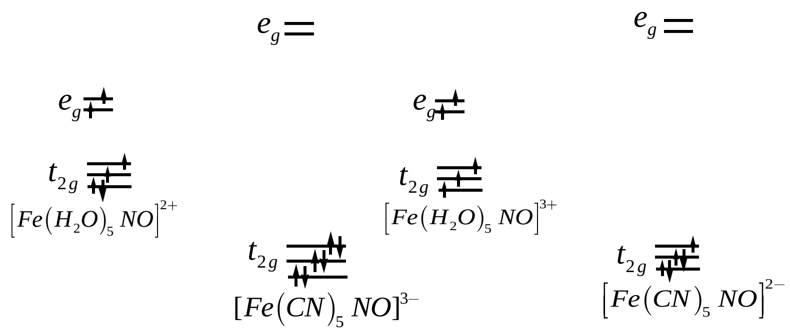
66. Hydration reaction.

67. The electron affinity values are in the order fluorine > oxygen > carbon.

68. The gas is less compressible (vander waals equation)

69. $CaC_2 + H_2O \rightarrow C_2H_2 \xrightarrow{H_2SO_4, HgSO_4} CH_3 + CHO$

70.



In $\left[Fe\left(H_2O\right)_5 NO\right]^{2+}$ and $\left[Fe\left(H_2O\right)_5 NO\right]^{3+}$, the unpaired electron present in higher energy anti bonding MO of NO transfers to lower energy t_{2g} orbital of Fe Still these contain unpaired electrons and are paramagnetic In $\left[Fe\left(CN\right)_5 NO\right]^{3-}$, the unpaired electron in higher. Energy anti bonding MO of NO cannot go into the higher energy eg orbital of Fe as there is no vacancy in lower energy t_{2g} orbital. So it remain in NO and as a whole the complex is paramagnetic. In $\left[Fe\left(CN\right)_5 NO\right]^{2-}$ the electron from NO can be transferred to vacant lower energy t_{2g} orbital. Since all the electrons are paired in this complex it is diamagnetic .

71. (1) $2MnO_4^- + 5H_2O_2 + 6H^+ \rightarrow 2Mn^{2+} + 5O_2 + 8H_2O$

22,400 ml of $O_2 \equiv$ 34 g of H_2O_2
224 ml of $O_2 \equiv$ 0.34 g of H_2O_2

Normality $= \frac{0.34}{17} \times \frac{1000}{20} = 1$

Volume strength = (5.6) (N) = 5.6 Vol

72. Two E and two Z isomers, one from methyl groups (Total Five)

73. Atomic sizes of both H and D are equal and hence H-H and D-D bond lengths are equal. The difference in H-H and D-D bond energies is not due to difference in bond lengths but due to difference in vibrational energies.

74. $H_2 / H^+ / I^- / I_2$

$H_2 + I_2 \rightarrow 2H^+ + 2I^-$

$0.7714 = 0.533 - \frac{0.059}{1} 10g\left(H^+\right)(0.1)$

75. $Zn-Hg / HCl$ with non polar solvent reduces double bond.

76. Magnesium slats gives precipitate with sodium bicarbonate only on boiling because $Mg\left(HCO_3\right)_2$ is soluble but on heating it decomposes to insoluble $MgCO_3$ and CO_2

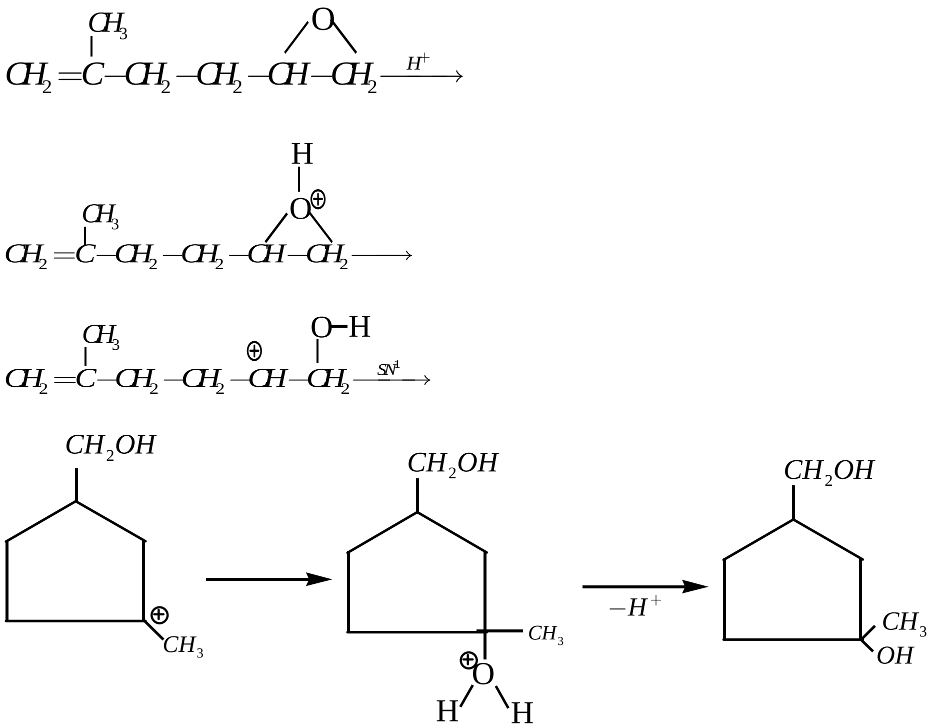
77. $\wedge_{eq} = X_1 + X_2 - X_3$

$\wedge_m = 2\left(x_1 + x_2 - x_3\right)$

$\wedge_m = \frac{K1000}{m}$

$S = \frac{10^3 x}{2\left(x_1 + x_2 - x_3\right)}$

$K_{sp} = S^2 = \frac{2.5x^2 10^5}{\left(x_1 + x_2 - x_3\right)^2}$

78. 

79. Among $(CH_3)_3 N \rightarrow O$ and $(CH_3)_3 P \rightarrow O$ $P \rightarrow O$ bond is stronger and shorter due to back bonding from oxygen to phosphorous $P \rightleftharpoons O$ thus polarity also decreases

80. Bredig's arc method

81. This is an example of intermolecular cannizzaro reaction.

82. IF_7 has zero dipolemoment

83. slow step will the rate determining step.

84. (1) Due to –I effect of -COOH ; oxalic acid is more acidic

85. Transition metals form ionic compounds in lower oxidation states while they form covalent compound in higher oxidation states .

86. (3) $i = 2\ (KCl \rightarrow K^+ + Cl^-)$

$\Delta T_b = iK_b m = 2 \times 0.52 \times 1 = 1.04$

$\therefore T_b = 101.04°C$

87. Rearrangement (In RDS migration is easy when donating groups present)

88. Except in the case of zinc, remaining three cases impurities are oxidized during the purification

89. (3) $\Delta G = \Delta H - T\Delta S$

At equilibrium $\Delta G = 0$

For a reaction to be non spontaneous ΔG should be positive

$\therefore T > T_e$

90. Resonance decreases basicity.

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