6. Assignment (Subjective Problems)

LEVEL – I

1. There are three different liquids (liquid 1, liquid 2 and liquid 3) having density 13.6 gm/cc., 1.3 gm/cc and 0.8 gm/cc. Liquid 1 is poured in a uniform U-tube, which is kept vertical. Liquid 2 & liquid 3 are poured separately into the two arms till upper surface of liquid 2 & 3 are same. What is the height of liquid 3 if the height of liquid 2 is 16cm?

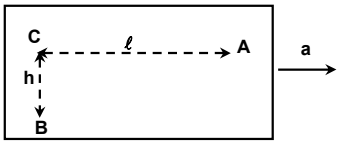
2. A very narrow hole exists at a height (H – h) from the bottom of a tank, which contains water upto a height H. Find the distance where stream of water coming out from the hole will strike the floor.

3. Two cylindrical vessels of radius r = 100m are filled with water upto heights H and 2H respectively. If the vessels are connected by a narrow tube through two holes made at their bottom, find the work done by gravity.

4. A garden hose having an internal diameter of 0.75 in. is connected to a lawn sprinkler that consists merely of an enclosure with 24 holes, each 0.050 in. in diameter. If the water in the hose has a speed of 3.0 ft/sec. at what speed does it leave the sprinkler holes?

5. A cube floating on mercury has one-fourth of its volume submerged. If enough water is added to cover the cube, what fraction of its volume will remain immersed in mercury? Does the answer depend on the shape of the body?

6. A seated tank containing a liquid of density moves with a horizontal acceleration 'a' as shown in the figure. Find the difference in pressure between the points A and B.



7. Find the work done in increasing the radius of a soap bubble from initial radius r_1 to final radius r_2. Given T = surface tension of soap solution.

8. If n identical water droplets falling under gravity with terminal velocity v coalesce to form a single drop which has the terminal velocity 4v. Find the number n.

9. A U-tube is partially filled with a liquid. The horizontal part of the tube is 2 m. The tube is accelerated horizontally with a constant acceleration of 5 m/s^2. What is the difference in the heights of the liquid in the two arms of the U-tube?

10. If the velocity gradient of water near the surface of a deep river is 6 s^1, find the shearing stress between the horizontal layers of water (coefficient of viscosity of water = 10^{-2} poise)

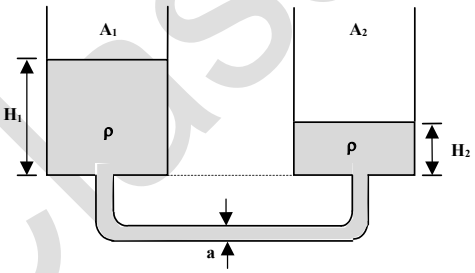
LEVEL – II

1. A silver block of mass 3.15kg is connected to a string and is then immersed in a liquid of relative density 0.82. Find the tension in the string, if relative density of silver is 10.5.

2. Water is flowing continuously from a tap having a bore of internal diameter 8×10^{-3} m. Calculate the diameter of the water stream at a distance 2×10^{-1} m below the tap. Assume that water velocity as it leaves the tap is 4×10^{-1} m/s.

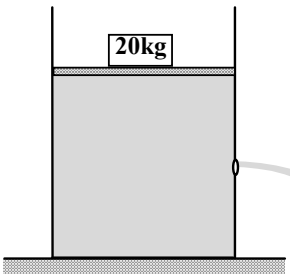
3. There are two identical small holes (cross section area = α) opposite sides of a tank containing a liquid. (density ρ) The tank is opened at the top. The difference is height between the two holes is 'h'. As the liquid comes out of the two holes, find the net horizontal force. The tank will experience.

4. Two cylindrical tanks of cross-sectional area A_1 and A_2 with their bases at the same level each contain a liquid of density ρ. The height of liquid in the tanks are H_1 and H_2, respectively. The tanks are joined together through a pipe of cross-sectional area 'a' as shown in the figure.



(a) Find the time taken to equalize the levels in the tanks.
(b) What is the work done by gravity in equalizing the levels?

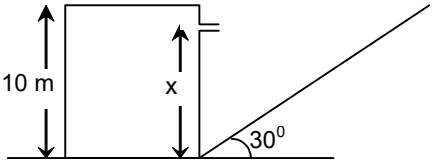
5. A long cylindrical tank of cross-section area $0.5m^2$ is filled with water. It has an opening at a height 50 cm from the bottom, having area of cross-section 1×10^{-4} m^2 . A movable piston of cross-section area almost equal to 0.5 m^2 is fitted on the top of the tank such that it can slide in the tank freely. A load of 20 kg is applied on the top of the water by piston, as shown in the figure. Find the speed of the water jet with which it hits the surface when piston is 1m above the bottom (Ignore the mass of the piston).



6. The vertical arms of U-tube have unequal radius R and r (R > r). If a liquid of surface tension T and density ρ rests in equilibrium inside the U-tube, find the level difference h between the meniscuses in the two arms.

7. A small hollow vessel which has a small hole in it is immersed in water to a depth of 40 cm before water enters into the vessel. Calculate the radius of the hole. [Surface tension of water = 70×10^{-3} N/m, density of water = 10^3 kg/m^3]

8. A rectangular tank of height 10 m filled with water, is placed near the bottom of an incline of angle 30^0. At height x from bottom a small hole is made (as shown in figure) such that the stream coming out from hole, strikes the inclined plane normally. Find x.



9. Water flows through a tapering horizontal tube of radii of cross section of the ends r_1 = 20 cm and r_2 = 10 cm. The velocity of water at the points for the radius of cross section r_1 is v_1 = 1m/sec. Find the force imparted by the emerging water at the other end of the tube.

10. Two soap bubbles of radii a and b combine under isothermal conditions to form a single bubble of radius c without any leakage of air. If P_o = atmospheric pressure and T = surface tension of soap solution, show that $P_o = \frac{4T\left(a^2+b^2-c^2\right)}{c^3-a^3-b^3}$

7. Assignment (Objective Problems)

LEVEL - I

1. A and B are two metallic pieces. They are fully immersed in water and then weighed. Now they show same loss of weight. The conclusion therefore is:
(A) A and B have same weight in air
(B) A and B have equal volumes
(C) The densities of the materials of A and B are the same
(D) A and B are immersed to the same depth inside water.

2. An ice cube contains a large air bubble. The cube is floating on the surface of water contained on a trough. What will happen to the water level, when the cube melts?
(A) It will remain unchanged (B) It will fall
(C) It will rise (D) First it will and then rise

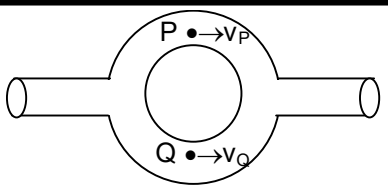
3. In a hydraulic lift, used at a service station the radius of the large and small piston are in the ratio of 20 : 1. What weight placed on the small piston will be sufficient to lift a car of mass 1500kg?
(A) 3.75Kg (B) 37.5Kg
(C) 7.5Kg (D) 75Kg

4. Water and mercury are filled in two cylindrical vessels upto same height. Both vessels have a hole in the wall near the bottom. The velocity of water and mercury coming out of the holes are v_1 and v_2 respectively. Thus
(A) $v_1 = v_2$ (B) $v_1 = 13.6v_2$
(C) $v_1 = v_2/13.6$ (D) $v_1 = \sqrt{(13.6)v_2}$

5. An ice cube contains a glass ball. The cube is floating on the surface of water contained in a trough on the surface of water contained in a trough. What will happen to the water level, when the cube melts?
(A) It will remain unchanged (B) It will fall
(C) It will rise (D) First it will fall and then rise

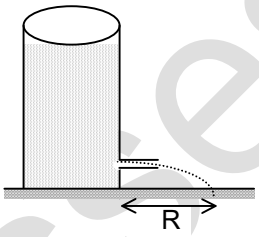
6. A square hole of side length ℓ is made at a depth of y and a circular hole is made at a depth of 4y from the surface of water in a water tank kept on a horizontal surface. If equal amount of water comes out of the vessel through the holes per second then the radius of the circular hole is equal to(r, $\ell << y$) :
(A) $\ell / \sqrt{2}$ (B) $\ell / 2$
(C) $\ell / \sqrt{\pi}$ (D) $\ell / \sqrt{2\pi}$

7. In the figure shown a liquid is flowing through a tube at the rate of 0.1 m^3/sec. The tube is branched into two semi circular tubes of cross sectional area A/3 and 2A/3. The velocity of liquid at Q is (the cross-section of the main tube (A) = 10^{-2} m^2 and V_P = 20 m/sec.):



(A) 5 m/sec (B) 30 m/sec
(C) 35 m/sec (D) None of these.

8. A small hole is made at a height of h′ = $(1/\sqrt{2})$ m from the bottom of a cylindrical water tank and at a depth of h = $\sqrt{2}$ m from the upper level of water in the tank. The distance, where the water emerging from the hole strikes the ground is:

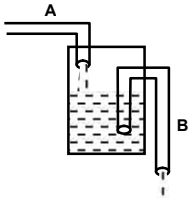


(A) $2\sqrt{2}$m (B) 1 m
(C) 2 m (D) None of these.

9. The excess pressure inside one soap bubble is three time that inside a second soap bubble. The ratio of the volumes of the two bubbles
(A) 1/9 (B) 9/1
(C) 1/27 (D) 27/1

10. An air bubble of diameter 2 mm rises steadily through a solution of density 1750 kg/m^3 at the rate of .35 cm/sec. Coefficient of viscosity of the solution is (Assume mass of the bubble to be negligible)
(A) 9 poise (B) 6 poise
(C) 11 poise (D) 4 poise

11. The velocity of the water flowing from the inlet pipe is less than the velocity of water flowing out from the spin pipe B.



(A) variation of water level in vessel will be irregular.
(B) water level will remains constant.
(C) the water level will perform periodic oscillation motions.
(D) none of the above.

12. In a streamline flow of a liquid
(A) every particle has its own velocity, different from others.
(B) all particles move with a constant velocity, even if the path is curvilinear.
(C) At a point on the streamline, particle can have two velocities.
(D) At a point on the streamline, particle can have only one velocity along the tangent.

13. A metallic sphere floats in an immiscible mixture of water ($\rho_w = 10^3$ kg/m^3) and a liquid (ρ_L = 13. 5×10^3 kg/m^3) such that 4/5 portion is in water and (1/5)th portion is in the liquid. The density of the metal in kg/m^3 is

(A) 4.5×10^3 (B) 4.0×10^3

(C) 3.5×10^3 (D) 3.0×10^3

14. A stream line body with relative density d_1 falls into air from a height h_1 on the surface of a liquid of relative density d_2, where d_2 is greater than d_1. The time of immersion of the body into the liquid will be

(A) $\sqrt{\left(\frac{2h_1}{g}\right)} \times \frac{d_1}{d_2 - d_1}$ (B) $\sqrt{\left(\frac{2h_1}{g}\right)}$

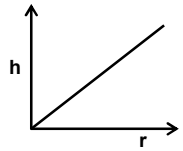
(C) $\sqrt{\left(\frac{2h_1}{g}\right)} \times \frac{d_1}{d_2}$ (D) $\sqrt{\left(\frac{2h_1}{g}\right)} \times \frac{d_2}{d_1}$

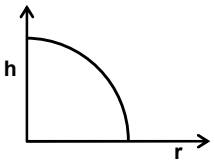
15. A tank is filled with water to a height H. Two holes are made on its side wall, one at a height of h from the bottom and other at a depth h from the top. The horizontal jets starting from the two holes meet the ground or side (in level with the bottom of the tank) at the same point. This distance of this point from the side of the tank is

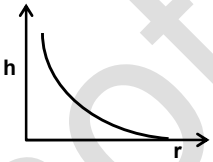
(A) $\sqrt{[4h(H-h)]}$ (B) $\sqrt{[h(H-h)]}$

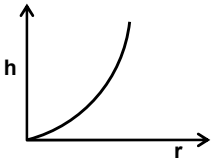
(C) $\sqrt{[2h(H-h)]}$ (D) $\sqrt{[3h(H-h)]}$

16. Which of the following graphs best represents the relation between the height h of the liquid in a capillary tube and radius of the capillary tube?

(A) 

(B) 

(C) 

(D) 

17. A boat floating in a tank is carrying passengers. If the passengers drink water, how will it affect the water level of the tank?
(A) It will go down
(B) It will rise
(C) It will remain unchanged
(D)It will depend on atmospheric pressure.

18. A cylinder is filled with non viscous liquid of density d to a height h_0 and a hole is made at a height h_1 from the bottom of the cylinder. The velocity of liquid issuing out of the hole is

(A) $\sqrt{(2gh_0)}$ (B) $\sqrt{2g(h_0 - h_1)}$

(C) $\sqrt{(dgh_1)}$ (D) $\sqrt{(dgh_0)}$

19. A spherical ball of mass m and radius r is allowed to fall in a medium of viscosity η. The time in which the velocity of the body increases from zero to 0.63 times the terminal velocity is called time constant (τ). Dimensionally τ can be represented by

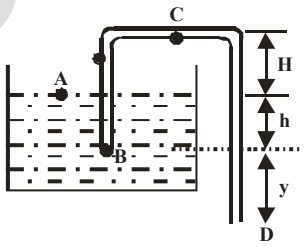
(A) $\frac{mr^2}{6\pi\eta}$ (B) $\sqrt{\frac{6\pi mr\eta}{g^2}}$

(C) $\frac{m}{6\pi\eta r}$ (D) none of these.

20. A large bottle is fitted with a capillary siphon. Ratio of times taken to empty the bottle when it is filled with (i) water (ii) petroleum of relative density 0.8.
(η_{water} = 0.001poise, η_{water} = 0.002 poise, d_{water} = 1000 kg/m^3)
(A) 5/4 (B) 4/5
(C) 2/5 (D) 3/5

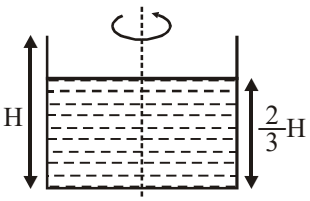
LEVEL - II

1. A Siphon tube is used to remove liquid from a container as shown in figure. In order to operate the Siphon tube it must initially be filled with the liquid.



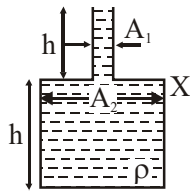
(A) Speed of the liquid through the Siphon is $\sqrt{2g(h+y)}$
(B) Pressure at C is $= P_{atm} - \rho g(h+H+y)$
(C) Pressure at P_A is less then P_D
(D) Pressure at P_A equal to P_D

2. A circular cylinder of radius R and height H is filled with water to a height $\frac{2}{3}$H. It starts rotating about its axis with constantly increasing angular speed. Choose the correct alternatives.



(A) at all speeds, shape of the free surface is parabolloid
(B) the free surface touches first the brim of cylinder and then the base of the cylinder
(C) the free surface cannot touch the base without spilling water
(D) the free surface touches the brim as well as base at the same instant

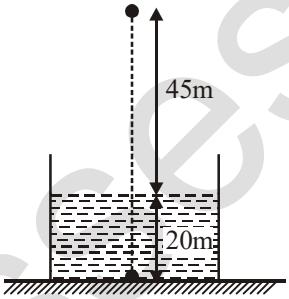
3. The vessel shown in the figure has two sections of areas of cross–section A_1 and A_2. A liquid of density ρ fills both the sections, up to a height h in each. Neglect atmospheric pressure



(A) the pressure at the base of the vessel is $2h\rho g$
(B) the force exerted by the liquid on the base of the vessel is $2h\rho gA_2$
(C) the weight of the liquid is < $2h\rho gA_2$
(D) the walls of the vessel at the level X exert a downward force $h\rho g\,(A_2 - A_1)$ on the liquid

4. A small body of density ρ' is dropped from rest at a height h into a lake of density ρ, where $\rho > \rho'$. Which of the following statement or statements is or are correct if all dissipative effects are neglected? (neglect viscosity)
   (A) the speed of the body just entering the lake is 2gh
   (B) the body in the lake experiences upward acceleration equal to $\{(\rho/\rho') - 1\}$ g
   (C) the maximum depth to which the body sinks in the lake is $h\rho'/(\rho - \rho')$
   (D) the body does not come back to the surface of the lake

5. A target object is placed at the bottom of a tank of depth 20 m, and filled with a liquid of density 500 gm/cc. Another object of density 100 gm/cc is dropped onto it, from a height of 45 m from the upper surface of the liquid. (take g = 10 m/s^2)
   (A) The second object will collide with the target.
   (B) The second object does not collide with the target.
   (C) The least separation between object and target is 11.25 m.
   (D) The retardation of the object in the liquid is 40 ms^{-2}.



6 A closed vessel is half filled with water. There is a hole near the top of the vessel and air is pumped out from this hole.
   (a) The water level will rise up in the level
   (b) The pressure at the surface of the water will decrease
   (c) The force by the water on the bottom of the vessel will decrease
   (d) The density of the liquid will decrease

7 In a streamline flow,
   (a) the speed of a particle always remains same
   (b) the velocity of a particle always remain same
   (c) the kinetic energies of all the particle arriving at a given point are the same
   (d) the momenta of all the particles arriving at a given point are the same

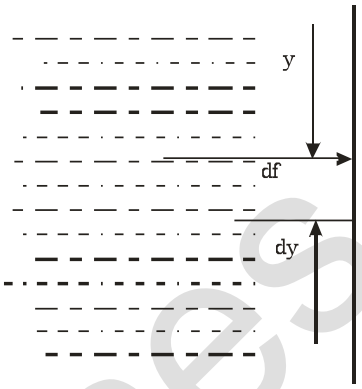
8 Water is flowing is streamline motion through a tube with its axis horizontal. Consider two points A and B in the tube at the same horizontal level.
   (a) The pressure at A and B are equal for any shape of the tube
   (b) The pressures are never equal
   (c) The pressure are equal if the tube has a uniform cross-section
   (d) The pressure may be equal if tube has non-uniform cross-section

9 There is a small hole near the bottom of an open tank filled with a liquid. The speed of the water ejected does not depend upon
   (a) area of the hole
   (b) density of the liquid
   (b) height of the liquid from the hole
   (d) acceleration due to gravity

10. Water is flowing through a long horizontal tube. Let PA and PB be the pressures at two points A and B of the tube
   (a) PA may be equal to PB
   (b) PA may be greater than PB
   (c) PA may be smaller than PB
   (d) PA = PB only if the cross-sectional area at A and B are equal

COMPREHENSION

I. Hydrostatic force on a submerged curved surface : consider first a vertical flat surface of height h and width W.



The pressure at a depth 'y' below the liquid surface

$P = \rho g y$ where ρ : density of liquid

$\therefore$ $dF = (\rho g y)(W dy)$

$= \rho g W y\, dy$

$\therefore$ $F = \int dF = \int_0^h \rho g W y\, dy$

$= \frac{1}{2}\rho g W h^2$

This result may be interpreted as

$F = (Wh)\left(\rho g \frac{h}{2}\right)$

= Projected area × pressure at the centroid of the projected area

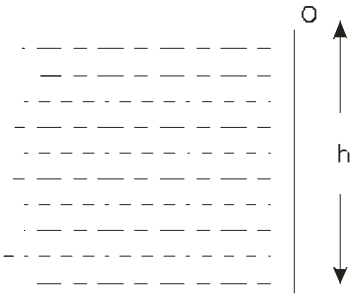
Projected area can be considered as the image of the surface (flat or curved) on a screen.

Thus,

F_X = Force in X-direction

= Projected area normal to X-direction × pressure at the centroid of the projected area

1. For the flat surface discussed in the passage, the total torque of the hydrostatic force about point 'O' is



(A) $\frac{2}{3}\rho g\, wh^3$ (B) $\frac{1}{3}\rho g\, wh^3$

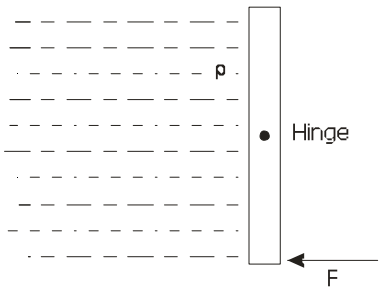
(C) $\rho g\, wh^3$ (D) $\frac{1}{2}\rho g\, wh^3$

2. The line of action of the resultant horizontal force acts at a distance r below point 'O'. The value of r is

(A) $\frac{h}{2}$ (B) $\frac{h}{3}$

(C) $\frac{2h}{3}$ (D) $\frac{h}{b}$

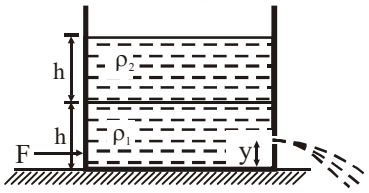
3. A flat plate of height 1m and width 1m is hinged at its middle as shown in the figure. To the left of the plate is a liquid of density ρ. To prevent the rotation of the plate, a force F is applied from the other side at the bottom. The value of F is



(A) Zero (B) $\frac{\rho g}{2}$

(C) $\frac{\rho g}{4}$ (D) $\frac{\rho g}{6}$

II. A cylindrical tank having cross–sectional area A = 0.5 m^2 is filled with two liquids of density ρ_1 = 900 kg m^{-3} and ρ_2 = 600 kg m^{-3}, to a height h = 60 cm each as shown in figure. A small whole having area a = 5 cm^2 is made in right vertical wall at a height y = 20 cm from the bottom. A horizontal force F is applied on the tank to keep it in static equilibrium. The tank is lying on a horizontal surface. Neglect mass of cylindrical tank comparison to mass of liquids. (take g = 10 ms^{-2})



4. The velocity of efflux
(A) 10 ms^{-1} (B) 20 ms^{-1}
(C) 4 ms^{-1} (D) 35 ms^{-1}

5. Horizontal force F to keep the cylinder in static equilibrium, if it is placed on a smooth horizontal plane
(A) 7.2 N (B) 10 N
(C) 15.5 N (D) 20.4 N

6. Minimum and maximum values of F to keep the cylinder in static equilibrium just after the water starts to spill through the hole. If the co-efficient of static friction between contact surfaces is 0.01.
(A) 0, 40 N (B) 5.4 N, 52.2 N
(C) 0, 70 N (D) 0, 52.2 N

MATCH THE FOLLOWING

1. With regard to dependence of quantities given in column a on the quantities mentioned in column b, match columns a and column b.

column I		Column II	
(A)	Young's modulus of a substance	(p)	Depends on area of cross-section
(B)	Bulk modulus of a substance	(q)	Depends on the nature of material
(C)	Modulus of rigidity of a substance	(r)	Depends on temperature
(D)	Volume of a substance	(s)	Depends on length

2. A cube of ice edge 4 cm is placed in an empty cylindrical glass of inner base area 64 cm^2. Assume that ice melts uniformly from each side so that it always remains its cubical shape then edge of the ice cube is x and height of water formed in cylindrical glass is h at the instant the ice cube just leaves contact with the bottom of the glass.
(density of ice = 0.9 gm/m/, density of water = 1 gm/m/). Then match the following :

Column I		Column II	
(A)	X	(p)	0.9 cm
(B)	H	(q)	1 cm
(C)	Volume of ice melted	(r)	56.7 cm^3
(D)	Volume of water formed	(s)	63 cm^3

8. HINTS (Subjective)

LEVEL- I

1. Pressure at A = Pressure at B $\Rightarrow$ $\rho_3 gh + \rho_1 g\ (H - h) = \rho_2 gH$

2. $H - h = \frac{1}{2} gt^2$ $\therefore$ $t = \{2(H - h)/g\}^{1/2}$

3. Calculate the change of potential energy of the system.

4. Apply continuity equation.

5. $v\sigma g = (v/4)\rho g$ $\Rightarrow$ $\sigma = \rho/4$

LEVEL- II

1. $T = mg - F_b = mg - V\rho g$

2. Applying Bernoulli 's theorem, we get

$\frac{1}{2}\rho v_1^2 + \rho gh = \frac{1}{2}\rho v_2^2$ (pressure being same)

3. Force = rate of change of momentum

4. $dh_2 = \frac{A_1}{A_2} dh_1$ (continuity equation)

The difference in levels decreases by

$dh = dh_1 + dh_2 = \left(\frac{A_1 + A_2}{A_2}\right) dh_1$

5. Pressure at the top is $P_1 = P_0 + \frac{20 \times 10N}{0.5m^2}$

9. Answers to the subjective assignment

LEVEL – I

1. 15.4 cm

2. $2\sqrt{(H-h)/h}$

3. 19.24 J

4. 29 ft/sec.

5. 0.19, NO

6. $P_B - P_A = h\rho g = \ell\rho a$

7. $8\pi T(r_2^2 - r_1^2)$

8. 8

9. 1 m.

10. 10^{-3} N/m^2

LEVEL – II

1. 29.04 N

2. 3.54×10^{-3} m

3. $2\alpha\rho gh$

4. (a) $\left[\frac{A_1A_2}{a(A_1+A_2)}\right]\sqrt{\frac{2(H_1-H_2)}{g}}$ (b) $\frac{1}{2}\rho g A_1A_2(H_1-H_2)^2$

5. 4.56 m/s

6. $\frac{2T}{\rho g}\left(\frac{1}{r}-\frac{1}{R}\right)$

7. 3.6×10^{-5} m

8. 8.33 m.

9. 502.65 N.

10. Answers to the objective assignment

LEVEL – I

1.	**B**	**2.**	**A**
3.	**A**	**4.**	**A**
5.	**B**	**6.**	**C**
7.	**D**	**8.**	**D**
9	**C**	**10.**	**A**
11.	**C**	**12.**	**D**
13.	**C**	**14.**	**A**
15.	**A**	**16.**	**C**
17.	**D**	**18.**	**D**
19.	**C**	**20.**	**C**

LEVEL – II

1.	**(A), (B), (D)**	2.	**(A), (B), (C)**
3.	**(A), (B), (C), (D)**	4.	**(B), (C)**
5.	**(B), (C), (D)**	6.	**(B), (C)**
7.	**(C), (D)**	8.	**(C)**
9.	**(A), (B)**	10.	**(A), (B), (C), (D)**

COMPREHENSION

1.	(B)	2.	(C)
3.	(D)	4.	(C)
5.	(A)	6.	(D)

MATCH THE FOLLOWING

1. (A) – (q), (r); (B) – (q), (r); (C) – (q), (r); (D) – (p), (r), (s)

2. (A) – (q); (B) – (p); (C) – (s); (D) – (r)