Subject	Topic	Lecture No.
Mathematics	Binomial Theorem, Mathematical logic	M-02

Class Work Solutions- Binomial Theorem

1. $\left(\sqrt{3}+1\right)^4+\left(\sqrt{3}-1\right)^4$

$=\left(\sqrt{3}\right)^4+{}^4C_1\left(\sqrt{3}\right)^3+{}^4C_2\left(\sqrt{3}\right)^2+{}^4C_3\left(\sqrt{3}\right)^3$

$+{}^4C_4\left(\sqrt{3}\right)^0+\left(\sqrt{3}\right)^4-{}^4C_1\left(\sqrt{3}\right)^3+{}^4C_2\left(\sqrt{3}\right)^2-{}^4C_3\left(\sqrt{3}\right)+{}^4C_4\left(\sqrt{3}\right)^0$

$=2\left[9+6(3)+1\right]=56=$ a rational number

2. Let x^4 occur in T_{r+1}

$T_{r+1}={}^{10}C_r\left(\frac{x}{2}\right)^{10-r}\left(-\frac{3}{x^2}\right)^r={}^{10}C_r\frac{x^{10-3r}(-3)^r}{(2)^{10-r}}$ $\quad\therefore 10-3r=4\Rightarrow 3r=10-4=6\therefore r=2$

$\therefore$ co-efficient of $x^4={}^{10}C_2\frac{(-3)^2}{2^8}=\frac{10\times9\times9}{1\times2\times256}=\frac{405}{256}$

3. $T_7=T_{6+1}={}^9C_6\left(\frac{4x}{5}\right)^3\left(-\frac{8}{5x}\right)^6$

Power of $x=3-6=-3$

4. $T_{r+1}={}^9C_r3^{9-r}(ax)^r$; co-efficient of $x^2={}^9C_23^7a^2$

Co-efficient of $x^3={}^9C_3(3)^6a^3$ $\quad\Rightarrow{}^9C_23^7a^2={}^9C_33^6a^3\Rightarrow\frac{9\times8}{1\times2}(3)=\frac{9\times8\times7}{1\times2\times3}a\Rightarrow a=\frac{9}{7}$

5. Co-efficient of $T_{2r+1}={}^{43}C_{2r}$, $\quad$ Co-efficient of $T_{r+2}={}^{43}C_{r+1}$

$\therefore{}^{43}C_{2r}={}^{43}C_{r+1}$ $\quad\therefore$ either $2r=r+1$ or $2r+r+1=43$

$\therefore$ either $r=1$ *or* $3r=42$ *i.e.* $r=14$

But $r\neq1$ $\quad[\because \text{for } r=1,\ 2r+1=r+2]$ $\quad\therefore r=14$

6. $T_5=24T_3\Rightarrow{}^{11}C_4x^4=24.{}^{11}C_2x^2\Rightarrow\frac{11\times10\times9\times8}{1\times2\times3\times4}x^4=24\frac{11\times10}{1\times2}x^2$

$\Rightarrow\frac{9\times8}{3\times4}x^2=24\Rightarrow x^2=\frac{24\times3\times4}{9\times8}=4$

7. $T_{r+1}={}^9c_r\left(\frac{3}{2}x^2\right)^{9-r}\left(-\frac{1}{3x}\right)^r={}^9c_r\left(\frac{3}{2}\right)^{9-r}x^{18-2r-r}\left(-\frac{1}{3}\right)^r$

$\therefore 18-3r=0\Rightarrow r=6$ $\quad\therefore$ Required term $=\frac{9\times8\times7}{1\times2\times3}.\frac{3^3}{8}\times\frac{1}{3^6}=\frac{3\times7\times27}{2\times3^6}=\frac{7}{18}$

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8. $T_{r+1} = {}^{12}c_r (2x)^{12-r}\left(-\frac{1}{2x^2}\right)^r \quad = {}^{12}c_r (2)^{12-r} x^{12-r-2r}\left(-\frac{1}{2}\right)^r$

$\therefore 12-3r=0 \Rightarrow 3r=12 \Rightarrow r=4 \qquad \therefore$ required term $= {}^{12}c_4 (2)^8\left(-\frac{1}{2}\right)^4 = {}^{12}c_4 2^4$

9. Middle term in $(1+x)^{2n} = T_{n+1} = {}^{2n}c_n x^n = \frac{2n!}{n!n!}$

$= \frac{[1.3.5...(2n-1)][2.4.6...2n]}{n!n!} = \frac{[1.3.5...(2n-1)]2^n}{n!}x^n$

10. Middle term $= T_3 = T_{2+1} = {}^4c_2 (3x)^2 (2)^2$

$\therefore$ co-efficient $= 6\times 9\times 4 = 216$

11. The sum of the odd binomial co-efficients of order $n = 2^{n-1}$ $\therefore$ for $(1+x)^{50}$, $\text{sum} = 2^{49}$

12. $T_{r+1} = {}^{10}C_r \left(\sqrt{\frac{x}{3}}\right)^{10-r}\left(\frac{3}{2x^2}\right)^r = {}^{10}C_r \frac{3^r}{3^{\left(\frac{10-r}{2}\right)}2^r} x^{\frac{10-r}{2}-2r}$

Since the term is independent of x $\therefore \frac{10-r}{2} - 2r = 0 \Rightarrow 10-r-4r=0 \Rightarrow r=2$

$\therefore$ required term $= T_3 = {}^{10}C_2 . \frac{3^2}{3^4.2^2} = \frac{10.9}{1.2}\times\frac{9}{81\times 4} = \frac{5}{4}$

13. ${}^nP_4 = 24 {}^nC_5 \Rightarrow n(n-1)(n-2)(n-3) = 24.\frac{n(n-1)(n-2)(n-3)(n-4)}{1.2.3.4.5}$

$\Rightarrow 1 = \frac{24}{120}(n-4) \Rightarrow n-4=5 \Rightarrow n=9$

14. $(1+x-3x^2)^{3148} = a_0 + a_1x + a_2x^2 + ...$ Putting $x=1$, we get

$(-1)^{3148} = a_0 + a_1 + a_2 .... \Rightarrow a_0 + a_1 + a_2 + ... = -1$

15. $T_{r+1} = {}^nC_r (x^2)^{n-r}\left(\frac{2}{x}\right)^r = {}^nC_r x^{2n-3r} 2^r \Rightarrow 2n-3r=0 \Rightarrow n = \frac{3r}{2}$

This is independent of x is a positive integer $\therefore$ $n=18$

16. $T_2 = {}^{2n}C_1 x; T_3 = {}^{2n}C_2 x^2; T_4 = {}^{2n}C_3 x^3$

$\therefore 2.{}^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3 \Rightarrow 2.\frac{2n(2n-1)}{2} = 2n + \frac{2n(2n-1)(2n-2)}{1.2.3} \Rightarrow 2n-1 = 1+\frac{(2n-1)(2n-2)}{6}$

$\Rightarrow 12n-12 = 4n^2-6n+2 \Rightarrow 4n^2-18n+14=0$ or $2n^2-9n+7=0$

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17. $(1+x)^m(1-x)^n=\left(1+mx+\frac{m(m-1)}{2}x^2+...\right)\quad\left(1-nx+\frac{n(n-1)}{2}x^2+...\right)$

Co-efficient of $x=(m-n)=3$ (given)

Co-efficient of $x^2=\frac{m(m-1)}{2}-mn\quad+\frac{n(n-1)}{2}=-6$ (given)

$\Rightarrow m^2-m-2mn+n^2-n=-12\Rightarrow m^2+n^2-2mn-(m+n)=-12$

Also, $m-n=3....(2)$. By (1) and (2)

$9-(m+n)=-12\therefore m+n=21....(3)$

$(2)+(3)$ gives $2m=24\therefore\quad m=12$

18. Co-efficient of middle terms in $(1+ax)^4$ and $(1-ax)^6$ are ${}^4C_2a^2,-{}^6C_3a^3$

By the given result ${}^4C_2a^2=-{}^6C_3a^3\Rightarrow 6a^2=-20a^3\quad\Rightarrow a=\frac{-6}{20}=\frac{-3}{10}$

19. $\because {}^nC_1+{}^nC_3+{}^nC_5+...={}^nC_0+{}^nC_2+{}^nC_4+...$

$\therefore {}^nC_0-{}^nC_1+{}^nC_3+....=0$

20. Co-efficient of $x^{17}=-(1+2+3+....+18)=-\frac{18}{2}(1+18)=-9\times19=-171$

21. $$\frac{C_1}{C_0}+2\frac{C_2}{C_1}+3\frac{C_3}{C_2}+....+\frac{nC_n}{C_{n-1}}=\frac{n}{1}+2\frac{\frac{n(n-1)}{2}}{n}+\frac{\frac{n(n-1)(n-2)}{3!}}{\frac{n(n-1)}{2!}}+....+\frac{n.1}{n}$$

$=n+(n-1)+(n-2)+...+1=\frac{n(n+1)}{2}\qquad\therefore K=2$

22. $C_0^2+C_1^2+....+C_n^2={}^{2n}C_n$

23. $(1-x+x^2)^n=a_0+a_1x+...+a_{2n}x^{2n}$ putting $x=1$ on both sides, we get $1=a_0+a_r+...+a_{2n}$

Putting $x=-1$ on both sides, we get $3^n=a_0-a_1+a_2-a_3+...+a_{2n}\quad(i)+(ii)$ gives

$3^n+1=2[a_0+a_2+a_4+....+a_{2n}]\qquad\therefore\frac{3^n+1}{2}=a_0+a_2+a_4+....+a_{2n}$

Home Work Solutions:

1. "The sun is a star," is a statement
2. p : we control population, q : we prosper

$\therefore$ we have $p \to q$

Its negation is $\sim(p \to q)$ *i.e.* $p \wedge \sim q$ i.e., we control population but we do not prosper.

3. p: examination is difficult, q: I shall pass, r : I study hard

Given result is is : $p \to (r \to q)$

Now $\sim(r \to q) = r \wedge \sim q$

$\sim(p \to (r \to q)) = p \wedge (r \wedge \sim q)$

The examination is difficult and I study hard but I shall not pass.

4. Since $\sim(p \to q) = p \wedge \sim q$

$\sim(\sim p \to q) = \sim p \wedge \sim q$

Hence (d) is correct

5. (b) is clearly correct

6.

p	q	$\sim q$	$p \wedge \sim q$	$\sim(p \wedge \sim q)$	$p \to \sim(p \wedge \sim q)$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	F	T	T
F	F	T	F	T	T

Result is neither tautology nor contradiction. Hence (d) is correct.

7. $\sim(q \vee \sim(p \wedge r)) = \sim q \wedge (\sim(\sim(p \wedge r)) = \sim q \wedge (p \wedge r)$

8.

p	$\sim p$	$p \to \sim p$	$\sim p \to p$	$(p \to \sim p) \wedge (\sim p \to p)$
T	F	F	T	F
F	T	T	F	F

Clearly, $(p \to \sim p) \wedge (\sim p \to p)$ is a contradiction.

9. Contrapositive of $p \to q$ is $\sim q \to \sim p$

Converse of $\sim q \to \sim p$ is $\sim p \to \sim q$

10. Water freezes iff temperature is less than $0°$.

$\therefore (a)$ is true

11. A logical statement in any sentence which is (i) meaningful (ii) declarative (iii) unambiguous. The statement is either true or false or equivalently valid or invalid

In the above options a, b, c are correct. Only option (d) is not a proposition

12. Inverse is $\sim p \to \sim q$

$\therefore \sim\left[p \vee (q \to r)\right] \to \sim\left[q \wedge p\right] = \left[\sim p \wedge \sim(q \to r)\right] \to (\sim q \vee \sim p) = \left[\sim p \wedge (q \wedge \sim r)\right] \to (\sim q \vee \sim p)$

13. Contra positive is $\sim q \to \sim p$

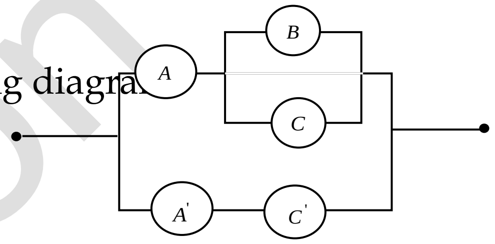
$\sim(q \wedge \sim r) \to \sim(\sim p \wedge q) = (-q \vee r) \to (p \vee \sim q)$

14.

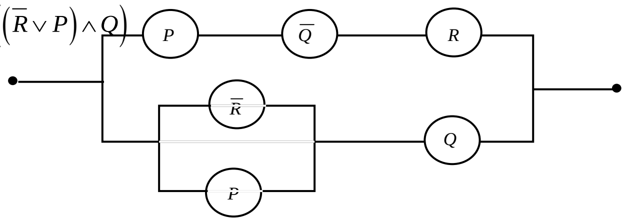
p	q	~p	~q	$p \wedge \sim q$	$(\sim p \vee q)$	$\sim(\sim p \vee q)$	$A \to B$
T	T	F	F	F	T	F	T
T	F	F	T	T	F	T	T
F	T	T	F	F	T	F	T
F	F	T	T	F	T	F	T

Given proposition is tautology

15. The $(A \wedge (B \wedge C)) \vee (A' \wedge C') \to$ By using diagram



16. By diagram, we have $(P \wedge \overline{Q} \wedge R) \vee ((\overline{R} \vee P) \wedge Q)$



17. The current will flow through the circuit only when p, q, r should always be closed.

18. p is false, q is true.

$\therefore p \vee q$ is $F \vee T = T$, $p \wedge q$ is $F \wedge T = F$, $p \vee q \to p \wedge q = T \to F = F$

19. By definition converse is $q \to p$

$\therefore$ if an integer is greater than 50 then it is less than 20

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20. Contrapositive $\sim q \to \sim p$

Let $p: x^2-5x+6=0$, $q: x=2$, $r: x=3$

By data $p \to (q \vee r)$

$\sim(q \vee r) \to \sim p = (\sim q \wedge \sim r) \to \sim p$

If $x \neq 2$ and $x \neq 3$ then $x^2-5x+6 \neq 0$

Home Work Solutions:

1. $T_{r+1} = {}^9C_r \left(\frac{x^2}{2}\right)^{9-r}\left(-\frac{2}{x}\right)^r \quad = {}^9C_r \frac{x^{18-2r}}{2^{9-r}} . \frac{(-2)^r}{x^r}$

$\therefore 18-3r=7 \Rightarrow 3r=11 \Rightarrow \; r=\frac{11}{3}$ not possible

$\therefore$ co-efficient of $x^7=0$

2. $T_{r+1} = {}^5C_r\left(x^2\right)^{5-r}\left(\frac{a}{x}\right)^r = {}^5C_r x^{10-2r-r}(a)^r$

$\therefore 10-3r=1 \Rightarrow 3r=9 \Rightarrow r=3$

$\therefore$ co-efficient. $x = {}^5C_3(a)^3 = 10a^3$

3. $T_{r+1} = {}^7C_r x^{7-r}(-2y)^r \quad \therefore 7-r=3 \Rightarrow r=4 \quad \therefore$ term contains $x^3 = T_{4+1} = T_5$

4. $T_{21} = T_{20+1} = {}^{44}C_{20}x^{20}$

$T_{22} = T_{21+1} = {}^{44}C_{21}(-x)^{21} = -{}^{44}C_{21}x^{21}$

$\therefore {}^{44}C_{20}x^{20} = -{}^{44}C_{21}x^{21}$

$\therefore x = -\frac{{}^{44}c_{20}}{{}^{44}c_{21}} = -\frac{44!}{24!20!} \times \frac{21!23!}{44!} = -\frac{21.20!23!}{20!24.23!} = -\frac{21}{24} = -\frac{7}{8}$

5. $T_{r+1} = {}^9C_r\left(y^{\frac{1}{6}}\right)^{9-r}\left(-y^{\frac{1}{3}}\right)^r$

$= {}^9C_r y^{\frac{9-r}{6}}.(-1)^r y^{\frac{-r}{3}} \qquad \therefore \frac{9-r}{6}-\frac{r}{3}=0 \Rightarrow 9-r-2r=0$

$\Rightarrow 3r=9 \Rightarrow r=3 \qquad \therefore$ required term $= {}^9C_3(-1)^3 = -{}^9C_3 = \frac{9\times8\times7}{1\times2\times3} = -3\times4\times7 = -84$

6. $T_{r+1} = {}^{18}C_r x^{18-r}\left(-\frac{3}{x^2}\right)^r = {}^{18}C_r x^{18-r-2r}(-3)^r.$

$\therefore 18-3r=0 \Rightarrow r=6 \quad \therefore$ required term $={}^{18}C_6(-3)^6={}^{18}C_6(3)^6$.

7. Middle term $=T_{5+1}={}^{10}C_5\left(\frac{1}{x}\right)^5={}^{10}C_5\frac{1}{x^5}$

8. The co-efficient of $(r+1)^{th}$ is ${}^nC_r$ But ${}^{18}C_{2r+3}={}^{18}C_{r-3}$

$\therefore 2r+3+r-3=18 \Rightarrow 3r=18 \Rightarrow r=6$

9. ${}^nC_{12}={}^nC_6 \Rightarrow n=12+6=18$

${}^nC_2={}^{18}C_2=\frac{18\times 17}{2}=9\times 17=153$

10. $\sum_{r=0}^{100} {}^{100}C_r(x-3)^{100-r}2^r=(x-3+2)^{100}=(x-1)^{100}$

$=x^{100}+{}^{100}c_r x^{99}(-1)^1+{}^{100}c_2x^{98}(-1)^2+....+{}^{100}c_{47}x^{53}(-1)^{47}$

Co-efficient of $x^{53}=-{}^{100}c_{47}=-{}^{100}c_{53}$

11. Let x^{-9} occur in T_{r+1}

Now $T_{r+1}={}^9C_r\left(\frac{x^2}{2}\right)^{9-r}\left(-\frac{2}{x}\right)^r \quad ={}^9C_r\frac{x^{18-3r}}{2^{9-2r}}(-1)^r$

Since x^{-9} occur in $T_{r+1} \quad \therefore 18-3r=-9 \quad \Rightarrow 3r=27 \quad \Rightarrow r=9$

$\therefore$ required co-efficient $=\frac{{}^9C_9(-1)^9}{2^{-9}}=-2^9=-512$

12. $(1+x^2)^5(1+x)^4=(1+{}^5c_1x^2+{}^5c_2x^4+{}^5c_3x^6+{}^5c_4x^8+{}^5c_5x^{10})$

$\times(1+{}^4c_1x+{}^4c_2x^2+{}^4c_3x^3+{}^4c_4x^4)$

Co-efficient of $x^5={}^5c_1.{}^4c_3+{}^4c_1.{}^5c_2=5\times 4+4\times 10=60$

13. $\left(\frac{1}{x}+1\right)^n(1+x)^n=\frac{1}{x^n}(1+x)^{2n}$

$=\frac{1}{x^n}(1+{}^{2n}C_1x+{}^{2n}C_2x^2+...+{}^{2n}C_{n-1}x^{n-1}+{}^{2n}C_nx^n+{}^{2n}C_{n+1}x^{n+1}+...+{}^{2n}C_{2n})$

Co-efficient of $\frac{1}{x}={}^{2n}C_{n-1}$

14. ${}^{14}C_1+{}^{14}C_3+...+{}^{14}C_{11}+{}^{14}C_{13}=\frac{2^{14}}{2}=2^{13}$

$\therefore {}^{14}C_1+{}^{14}C_3+...+{}^{14}C_{11} \quad =2^{13}-{}^{14}C_{13}=2^{13}-14 \;=2^{13}-14$

15. $^{13}C_0 + ^{13}C_1 + ^{13}C_2 + ... + ^{13}C_{13} = 2^{13}$

$\therefore ^{13}C_2 + ^{13}C_3 + ... + ^{13}C_{13} = 2^{13} - 1 - 13 = 2^{13} - 14$

16. Let $199 = T_n$ of $1,3,5...$ $\quad = 1 + (n-1)2 = 2n - 1 \Rightarrow 2n = 200 \Rightarrow n = 100$

Co.efficient of x^{99} $in\ (x+1)(x+3)...(x+199) = (1+3+5+.....+199)$

17. $C_0 + ^3C_1 + ^5C_2 + ... + (2n+1)C_n = (C_0 + C_1 + ... + C_n) + ^2C_1 + ^4C_2 + ... + 2nC_n$

$$= 2^n + 2(C_1 + 2C_2 + ... + n.C_n) = 2^n + 2\left(n + 2.\frac{n(n-1)}{2!} + 3\frac{n(n-1)(n-2)}{3!} + ... + n.1\right)$$

$$= 2^n + 2\left(n + n(n-1) + \frac{n(n-1)(n-2)}{2!} + ... + n\right) = 2^n + 2n\left(1 + (n-1) + \frac{(n-1)(n-2)}{2!} + ... + 1\right)$$

$$= 2^n + 2n\left(^{n-1}C_0 + ^{n-1}C_1 + ^{n-1}C_2 + ^{n-1}C_3 + ... + ^{n-1}C_{n-1}\right) = 2^n + 2n.2^{n-1} = 2^n + n.2^n = 2^n(n+1)$$

18. $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$ Also $(x+1)^n = C_n + C_{n-1}x + C_{n-2}x^2 + ... + C_0x^n$

Multiply both sides and comparing co-efficient of x^{n-r} on both sides, we get

$$C_0C_r + C_1C_{r+1} + ... + C_{n-r}C_n \quad = ^{2n}c_{n-r} = \frac{2n!}{(n-r)!(n+r)!}$$

19. $(x+y)^{100} + (x-y)^{100} = 2\left[x^{100} + ^{100}C_2x^{98}y^2 + ^{100}C_4x^{96}y^4 + ... + ^{100}C_{100}y^{100}\right]$

Number of terms $= 51$ (clearly)

20. $S = 2^n + n2^{n-1}$

Here $2^n + n.2^{n-1} = 576 \Rightarrow 2^{n-1}(n+2) = 64 \times 9 = 2^6(2+7) = 2^{7-1}(2+7)$ $\qquad \therefore n = 7$

21. Total number of terms in $(2x+y+3z)^{10} = ^{10+3-1}C_{3-1} = ^{12}C_2 = \frac{12 \times 11}{1 \times 2} = 66$

22. p: It rains, q: I shall go to school

Thus, we have $p \to q$

Its negation is $\sim(p \to q)$ *i.e.* $p \wedge \sim q$

i.e. It rains and I shall not go to school

23. $\sim(p \wedge q) = -p \vee \sim q$

24. $\sim(p \vee (\sim q)) = \sim p \wedge \sim(\sim q) = \sim p \wedge q$

25. $\sim(p \leftrightarrow q) = (p \wedge \sim q) \vee (q \wedge \sim p)$

26. $p \to q$ is false only when p is true and q is false. $\therefore p \to q$ is false when p is true and $q \vee r$ is false, and $q \vee r$ is false when both q and r are false. Hence truth values of p, q, r are respectively T, F, F.

27. Mathematics is interesting is not a logical sentence. It may be interesting for some persons are may not be interesting for others. $\therefore$ this is not a propositions.

28. Inverse of $p \to \sim q$ is $\sim p \to q$

Contrapositive of $\sim p \to q$ is $\sim q \to p$

29. (b) is true

30. The contrapositive is $\sim[\sim q \to \sim r] \to \sim p$ i.e. $\sim[\sim(\sim q) \vee \sim r] \to \sim p$ $[\because p \to q = \sim p \vee q]$

i.e. $\sim q \wedge r \to \sim p$ i.e. $\sim[q \vee \sim r] \to \sim p$

31.

p	q	$p \to q$	$\sim(p \to q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$\sim(p \to q) \leftrightarrow \sim p \vee \sim q$
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	T	F	T	F
F	F	T	F	T	T	T	F

Last column shows that result is neither a tautology nor a contradiction

32.

p	q	$\sim q$	$p \wedge \sim q$	$\sim(p \wedge \sim q)$	$p \Rightarrow \sim(p \wedge \sim q)$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	F	T	T
F	F	T	F	T	T

Result is neither tautology nor contradiction.

33. $(p\wedge \sim q)\wedge(\sim p\wedge q)=(p\wedge \sim p)\wedge(\sim q\wedge q)=f\wedge f=f$

(By using associative laws and commutative laws)

$\therefore (p\wedge \sim q)\wedge(\sim p\wedge q)$ is a contradiction $\therefore (b)$ holds