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Mathematics	Binomial Theorem, Mathematical logic	M-02

Binomial Theorem

$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + ...+ {}^nC_r a^{n-r}b^r + ...+ b^n$ where n is a positive integer and ${}^nC_r = \frac{n!}{(n-r)!\,r!}$

Important features of the Binomial Theorem

1. There are $(n+1)$ terms in the expansion $(a+b)^n$.
2. Expansion of $(a+b)^n$ is a homogeneous polynomial of degree n in a and b.
3. Since ${}^nC_r = {}^nC_{n-r}$, the binomial coefficients are symmetric above the centre.
4. The $(r+1)^{\text{th}}$ term in the above expansion is $T_{r+1} = {}^nC_r a^{n-r}b^r$.
5. If the index is odd, the number of terms in the expansion is even and therefore there are two middle terms and if it is even the number of terms is odd and there is only one middle term.
6. The coefficients ${}^nC_0, {}^nC_1, {}^nC_2 ..... {}^nC_r$ are called the binomial coefficients of order n. Also binomial coefficients are denoted by $C_0, C_1, C_2, ...C_r ....C_n$ i.e. C_r means ${}^nC_r$.

Identities connected with Binomial Coefficients

If $C_0, C_1, C_2, ...C_n$ are binomial coefficients of the expansion $(1+x)^n$ then

1. $C_0 + C_1 + C_2 + ... + C_n = 2^n$ and $C_0 - C_1 + C_2 - ..... + (-1)^n C_n = 0$
2. $C_0 + C_2 + C_4 + ... = C_1 + C_3 + C_5 + ... = 2^{n-1}$
3. $C_0^2 + C_1^2 + C_2^2 + ... + C_n^2 = \frac{(2n)!}{(n!)^2}$
4. $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + ... + \frac{C_n}{n+1} = \frac{2^{n+1}-1}{n+1}$
5. $C_0C_r + C_1C_{r+1} + C_2C_{r+2} + ... + C_{n-r}C_n = \frac{(2n)!}{(n-r)!(n+r)!}$
6. $C_1 + 2C_2 + 3C_3 + ... + nC_n = n \cdot 2^{n-1}$

Time saving results (Short cut methods)

- The coefficient x^m in the expansion of $\left(ax^p + \frac{b}{x^q}\right)^n$ is the coefficient of $(r+1)^{th}$ term, where $r = \frac{np-m}{p+q}$.

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- The term independent of x in the expansion of $\left(ax^p + \frac{b}{x^q}\right)^n$ is the $(r+1)^{th}$ where $r = \frac{np}{p+q}$.
- If the coefficient of $r^{th}, (r+1)^{th}, (r+2)^{th}$ terms of $(1+x)^n$ are in A.P. then $n^2 - (4r+1)n + 4r^2 = 2$.
- Sum of the coefficients of $(ax+by)^n$ is $(a+b)^n$. The result obtained by putting $x=1,\ y=1$.
- In the expansion of $(x+y+z)^n$ the sum of all the coefficient can be obtained by putting $x=y=z=1$.
- If $n \in R$ & $|x|<1$, then $(1+x)^{-n} = 1 - nx + \frac{n(n-1)}{2!}x^2 - \frac{n(n-1)(n-2)}{3!}x^3 + ...$
- For $|x|<1,\ (1-x)^{-n} = 1 + {}^nC_1.x + {}^{n+1}C_2.x^2 + {}^{n+2}C_3.x^3 + ...$
- For $|x|<1,\ (1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - .....$
- For $|x|<1,\ (1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 - .....$
- For $|x|<1,\ (1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + 5x^4 - .....$
- For $|x|<1,\ (1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + 5x^4 - .....$
- For $|x|<1,\ (1+x)^{-3} = 1 - 3x + 6x^2 - 10x^3 + 15x^4 - .....$
- For $|x|<1,\ (1-x)^{-3} = 1 + 3x + 6x^2 + 10x^3 + 15x^4 - .....$
- In the expansion of $(x+a)^n$, $\frac{T_{r+1}}{T_r} = \frac{n-r+1}{r}.\frac{a}{x}$. i.e., the ratio of $(r+1)^{th}$ term to r^{th} term is $\frac{n-r+1}{r}.\frac{a}{x}$
- The greatest coefficient of $(1+x)^n$ is given by (i) ${}^nC_{\frac{n}{2}}$ if n is even & (ii) ${}^nC_{\frac{n-1}{2}}$ & ${}^nC_{\frac{n+1}{2}}$ if n is odd (both being equal).
- p^{th} term from the end in the binomial expansion of $(x+y)^n$ is $(n-p+2)^{th}$ term from the beginning.
- If A is the sum of the odd terms and B is the sum of even term in the expansion of $(x+a)^n$ then $A^2 - B^2 = \left(x^2 - a^2\right)^n,\ \ 4AB = (x+a)^{2n} - (x-a)^{2n}$.
- If $0 < r < n$ & n & r are natural numbers then

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(i) $\frac{{}^nC_r}{{}^nC_{r-1}}=\frac{n-r+1}{r}$ (ii) $\frac{{}^nC_r}{{}^nC_{r+1}}=\frac{r+1}{n-r}$ (iii) ${}^nC_{r-1}+{}^nC_r={}^{n+1}C_r$

(iv) ${}^nC_x={}^nC_y\Rightarrow x=y$ or $x+y=n$ (v) ${}^nC_r={}^nC_{n-r}$

- The coefficient of x^{n-1} in the expansion of $(x-1)(x-2).....(x-n)$ is $-\frac{n(n+1)}{2}$
- The coefficient of x^{n-1} in the expansion of $(x+1)(x+2).....(x+n)$ is $\frac{n(n+1)}{2}$
- If the coefficient of p^{th} & q^{th} terms in the expansion $(1+x)^n$ are equal then $p+q=n+2$.
- The number of terms in the expansion of $(x+a)^n-(x-a)^n=\frac{n}{2}$ is n is even and $\frac{n+1}{2}$ if n is odd.
- The number of terms in the expansion of $(x+a)^n+(x-a)^n=\frac{n+2}{2}$ is n is even and $\frac{n+1}{2}$ if n is odd.
- The number of terms in the expansion $(x_1+x_2+.....+x_n)^n={}^{(n+r-1)}C_{r-1}$
- $a.C_0+(a+d).C_1+(a+2d).C_2+.....+(a+nd).C_n=(2a+nd).2^{n-1}$
- Class Work Problems

1. $\left(\sqrt{3}+1\right)^4+\left(\sqrt{3}-1\right)^4$ is equal to

   (a) a rational number (b) an irrational number

   (c) a negative integer (d) none of these

2. The coefficient of x^4 in $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$ is

   (a) $\frac{405}{256}$ (b) $\frac{504}{259}$ (c) $\frac{450}{263}$ (d) none of these

3. The power of x occurring in the 7th term in the expansion of $\left(\frac{4x}{5}-\frac{8}{5x}\right)^9$ is

   (a) 3 (b) –3 (c) 5 (d) –5

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4. If the co-efficient of x^2 and x^3 in the expansion of $(3+ax)^9$ are the same, then the value of a is

(a) $\frac{9}{7}$ (b) $\frac{7}{9}$ (c) $-\frac{9}{7}$ (d) $-\frac{7}{9}$

5. In the expansion of $(1+x)^{43}$, co-efficient of the $(2r+1)th$ and the $(r+2)th$ term are equal, then r is equal to

(a) 7 (b) 14 (c) 21 (d) none of these

6. In the expansion of $(1+x)^{11}$, the fifth term is 24 times the third term. Then the value of x^2 is

(a) 4 (b) 9 (c) 16 (d) 24

7. The term independent of x in $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$ is

(a) $\frac{7}{18}$ (b) $\frac{5}{18}$ (c) $\frac{11}{18}$ (d) $\frac{13}{18}$

8. The term independent of x in the expansion of $\left(2x-\frac{1}{2x^2}\right)^{12}$ is

(a) ${}^{-12}C_3 2^6$ (b) ${}^{-12}C_5 2^2$ (c) ${}^{12}C_6.2^4$ (d) ${}^{12}C_2 2^4$

9. The middle term in the expansion of $(1+x)^{2n}$ is

(a) ${}^{2n}C_n$ (b) ${}^{2n}C_{n+1}x^{n+1}$ (c) ${}^{2n}C_{n-1}x^{n-1}$ (d) $\frac{1.3.5...(2n-1)}{n!}.2^n x^n$

10. In the expansion of $(3x+2)^4$, the coefficient of middle term is

(a) 81 (b) 54 (c) 216 (d) 36

11. Sum of the odd binomial coefficients of $(1+x)^{50}$ is

(a) 2^{50} (b) 2^{50-1} (c) 2^{49} (d) none of these

12. The coefficient of the term independent of x in the expansion of $\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$ is

(a) $\frac{9}{4}$ (b) $\frac{3}{4}$ (c) $\frac{5}{4}$ (d) $\frac{7}{4}$

13. If ${}^nP_4 = 24\,{}^nC_5$, then the value of $n =$

(a) 5 (b) 9 (c) 15 (d) 10

14. The sum of the co-efficients in the expansion of $(1+x-3x^2)^{3148}$ is

(a) 8 (b) 7 (c) 1 (d) –1

15. If the expansion of $\left(x^2+\frac{2}{x}\right)^n$ for positive integer n has a term independent of x, then n is

(a) 23 (b) 18 (c) 16 (d) 0

16. If co-efficients of 2nd, 3rd and 4th term in the expansion of $(1+x)^{2n}$ are in A.P. then

(a) $2n^2+9n+7=0$ (b) $2n^2-9n+7=0$ (c) $2n^2-9n-7=0$ (d) $2n^2+9n-7=0$

17. If in the expansion of $(1+x)^m(1-x)^n$, the co-efficients of x and x^2 are 3 and –6 respectively then m is

(a) 6 (b) 9 (c) 12 (d) 24

18. The co-efficient of the middle term in the binomial expansion in powers of x of $(1+ax)^4$ and of $(1-ax)^6$ is the same value of a equals

(a) $-\frac{3}{10}$ (b) $\frac{10}{3}$ (c) $-\frac{5}{3}$ (d) $\frac{3}{5}$

19. The value of ${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + ...(-1)^n\,{}^nC_n$ is

(a) 2^n-1 (b) n (c) 2^n (d) 0

20. The co-efficient of x^{17} in the expansion of $(x-1)(x-2)(x-3)...(x-18)$ is

(a) 342 (b) –171 (c) $\frac{171}{2}$ (d) 684

21. Let $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$ and $\frac{C_1}{C_0}+\frac{2.C_2}{C_1}+\frac{3.C_3}{C_2}+...+\frac{n.C_n}{C_{n-1}} = n\frac{(n+1)}{K}$ The value of K is

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{3}$ (d) 3

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22. If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, then $C_0^2 + C_1^2 + ... + C_n^2$ are equal to

(a) ${}^{2n}C_n$ (b) ${}^{2n}C_{n-1}$ (c) ${}^{2n+1}C_n$ (d) ${}^{2n-1}C_n$

23. If $(1-x+x^2)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$ then $a_0 + a_2 + a_4 + ... + a_{2n}$ equals

(a) $\frac{3^n - 1}{2}$ (b) $\frac{3^n + 1}{2}$ (c) $3^n + \frac{1}{2}$ (d) $3^n = \frac{1}{2}$

Answers

1. a	2. a	3. b	4. a	5. b	6. a	7. a	8. c	9. d	10.c
11.c	12.c	13.b	14.d	15.b	16.b	17.c	18.a	19.d	20.b
21.b	22.a	23.b							

Mathematical Logic

Proposition: - A proposition is a statement, which in a given context is either true or false but not both.

The truth or the falsity of a proposition is true is called its truth-value. If a proposition is true its truth value is denoted by 'T' and if it is false then its truth value is denoted by 'F'.

Example: 1) 5 is a prime number - It is proposition with truth value T.

2) The sun rises in the East - It is proposition with truth value F.

3) Good morning - It is not a proposition

Compound Proposition: The propositions connected by combining two or more propositions by using the phrases "not", "or"," and", "If - - - - then " and "If and only if" are called compound propositions.

We have to find the truth value of compound proposition by knowing the truth values of the simple proposition. The method of determining the truth value of a compound proposition is called the logical inference.

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1) **Negation:** A proposition obtained by adding the word "not" at an appropriate place in the given proposition is called negation of given proposition.

The negation of proposition $'p'$ is denoted by $\sim p$ and read as "not p".

p	$\sim p$
T	F
F	T

Ex: p : 4 is an even number.

Then is $\sim p$: 4 not an even number

Here p is true and $\sim p$ is false

2) **Conjunction:** A conjunction is a compound proposition obtained by combining the two proposition by using the word "and".

If p and q are two propositions then conjunction of p and q is denoted by $p \wedge q$ and is read as $"p \text{ and } q"$.

The truth table for $p \wedge q$ is given by

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Rule: $p \wedge q$ is true only if both p and q are true and is false in all other cases.

3) Disjunction: A disjunction is a compound proposition obtained by combining the two propositions by using the word 'or'.

If p and q are two propositions then the disjunction of p and q is denoted by $p \vee q$ and is read as $"p \text{ or } q"$.

The truth table for $p \vee q$ is

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Rule: $p \vee q$ is false only when both p and q are false and is true in all other cases.

4) Implication or Conditional: The conditional is a compound proposition obtained by connecting the two propositions by using the words "If - - - - - - then".

If p and q are two propositions then the conditional is "If p then q" and is denoted by $p \to q$ and is read as "p implies q" or "if p then q". Here 'p' is called hypothesis and q is called conclusion.

The truth table for $p \to q$ is

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Rule: $p \to q$ is false only if p is true and q is false and is true in all other cases.

5) **Biconditional or Equivalence:** A proposition obtained by combining two propositions by using the word "if and only if" is called biconditional or equivalence. If p and q are two propositions then biconditional of p and q is "p if and only if q" and is denoted by $p \leftrightarrow q$. The truth table for $p \leftrightarrow q$ is

p	q	$p \to q$
T	T	T
T	F	F
F	T	F
F	F	T

Rule: $p \leftrightarrow q$ is true when p and q are both true or both false and is false in other cases.

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Inverse, converse and Contra positive:

If $p \to q$ is any given implication then

(1) $\sim p \to \sim q$ is called its inverse

(2) $q \to p$ is called its converse

(3) $\sim q \to \sim p$ is called its contra positive.

The truth table is given by

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim q \to \sim p$	$q \to p$	$\sim p \to \sim q$
T	T	F	F	T	T	T	T
T	F	F	T	F	F	T	T
F	T	T	F	T	T	F	F
F	F	T	T	T	T	T	T

Tautology and Contradiction: A compound proposition is said to be tautology if it is always true for all possible combinations of truth values of its components.

A compound proposition is said to be a contradiction if it is always false for all possible combinations of truth values of its components.

Logical Equivalence: Two simple propositions p and q which have the same truth values are said to be logically equivalent and is denoted by $p \equiv q$.

Ex: $p : 3+4=7 \quad q : 10 > 6$

Here truth values of p and q are T and T

$\therefore \quad p \equiv q$

Two compound propositions a and b having the same components are said to be logically equivalent if their truth values are identical for each combination of the truth values of its components and is written as $a \equiv b$.

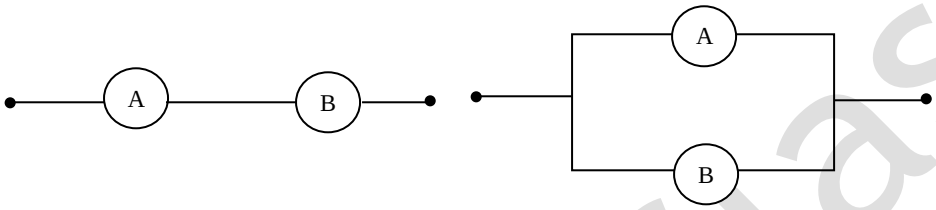
Note: 1) $p \to q \equiv \sim q \to \sim p$ 2) $(\sim p \to \sim q) \equiv q \to p$

Switching Circuits:

One of the applications of mathematical logic is to switching circuits. A switching network is an arrangement of wires and switches which connect two terminals. If a

switch is on (or closed) then it permits the flow of current and if a switch is off (or open) then it prevents the flow of current.

Basic series network and basic parallel network : If A and B are connected in series then it is denoted by $A \wedge B$ and if they are connected in parallel, it is denoted by $A \vee B$.



Class work Problems:

1. Which of the following is a statement?
   (a) May you live long! (b) May God bless you!
   (c) The sun is a star (d) Hurrah! We have won the match
2. Negation of the preposition: If we control population growth, we prosper.
   (a) If we do not control population growth, we prosper
   (b) If we control population, we do not prosper
   (c) We control population but we do not prosper
   (d) We do not control population, but we prosper
3. Negation of the compound proposition: If the examination is difficult, then I shall pass if I study hard
   (a) The examination is difficult and I study hard and I shall pass
   (b) The examination is difficult and I study hard but I shall not pass
   (c) The examination is not difficult and I study hard and I shall pass
   (d) None of these

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4. Which of the following is logically equivalent to $\neg(\neg p \to q)$?

   (a) $p \wedge q$ (b) $p \wedge \neg q$ (c) $\neg p \wedge q$ (d) $\neg p \wedge \neg q$

5. Which of the following is not logically equivalent to the proposition:

   "A real number is either rational or irrational".

   (a) If a number is neither rational nor irrational then it is not real

   (b) If a number is not a rational or not an irrational, then it is not real

   (c) If a number is not real, then it is neither rational nor irrational

   (d) If a number is real, then it is rational or irrational

6. The proposition $p \to \neg(p \wedge \neg q)$ is

   (a) contradiction (b) a tautology (c) either (a) or (b) (d) neither (a) nor

7. The negative of $q \vee \neg(p \wedge r)$ is

   (a) $\neg q \vee \neg(p \wedge r)$ (b) $\neg q \wedge (p \wedge r)$ (c) $\neg q \vee (p \wedge r)$ (d) $\neg q \vee (p \wedge r)$

8. The proposition $(p \to \neg p) \wedge (\neg p \to p)$ is a

   (a) Tautology and contradiction (b) Neither tautology nor contradiction

   (c) Contradiction (d) Tautology

9. The converse of the contrapositive of $p \to q$ is

   (a) $\neg p \to q$ (b) $p \to \neg q$ (c) $\neg p \to \neg q$ (d) $\neg q \to p$

10. p: Water froze this morning

    q: This morning temperature was $< 0°$. Thus

    (a) p and q are logically equivalent (c) p is the inverse of q

    (c) p is the converse of q (d) p is the contrapositive of q

11. Which one of the following is not a proposition?

    (a) $2^2 + 3^2 = 5^2$ (b) 2 is an even prime

(c) If $a^2 = b^2$ then $a = \pm b$ (d) Wish you happy birthday

12. The inverse of the compound proposition $[p \vee (q \to r)] \to (q \wedge p)$ is

(a) $[\sim p \wedge (q \wedge \sim r)] \to (\sim q \vee \sim p)$ (b) $[\sim p \wedge (\sim q \wedge -r)] \to (\sim q \vee \sim p)$

(c) $[\sim p \wedge (\sim q \wedge r)] \to (\sim q \vee \sim p)$ (d) $[\sim p \wedge (\sim q \vee \sim r)] \to (\sim q \vee \sim p)$

13. The contra positive of $(\sim p \wedge q) \to (q \wedge \sim r)$ is

(a) $(p \vee \sim q) \to (\sim q \vee r)$ (b) $(\sim q \vee r) \to (\sim p \vee q)$

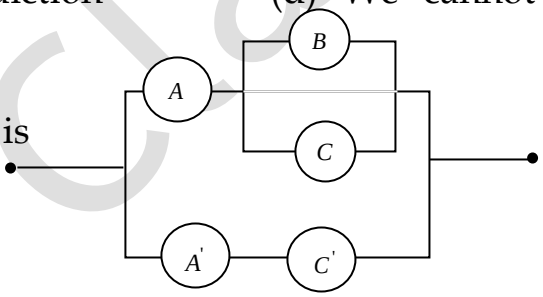
(c) $(\sim q \vee r) \to (p \vee \sim q)$ (d) $(\sim p \vee r) \to (\sim p \wedge \sim r)$

14. The compound proposition $(p \wedge \sim q) \to \sim (\sim p \vee q)$ is

(a) A contradiction (b) A tautology

(c) Neither a tautology nor a contradiction (d) We cannot come to any conclusion

15. The Boolean polynomial for the circuit is



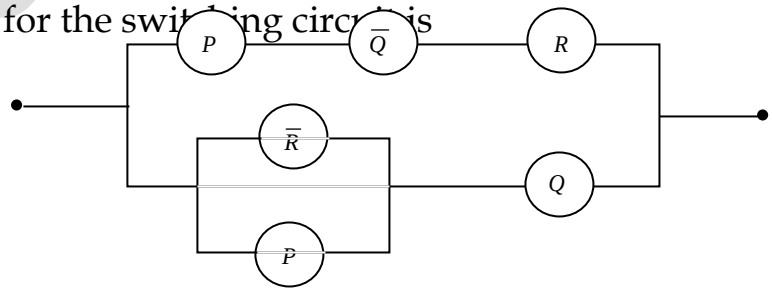
(a) $(A \vee (B \wedge C)) \wedge (A' \vee C')$

(b) $(A \wedge (B \vee C)) \vee (A' \wedge C')$

(c) $(A \vee (B \wedge C)) \vee (A' \vee C')$

(d) $(A \wedge (B \vee C)) \wedge (A' \wedge C')$

16. The Boolean polynomial for the switching circuit is



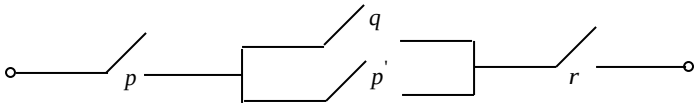
(a) $(P \wedge \overline{Q} \wedge R) \vee ((\overline{R} \wedge P) \vee Q)$

(b) $(P \wedge \overline{Q} \wedge R) \vee ((\overline{R} \vee P) \wedge Q)$

(c) $(P \wedge Q \wedge R) \vee ((\overline{R} \vee P) \wedge Q)$

(d) $(P \wedge \overline{Q} \wedge R) \vee ((\overline{R} \vee P) \wedge \overline{Q})$

17. When does the current flow through the following circuit?



(a) p,q,r should always be closed (b) p, q, r should be open

(c) always (d) None of these

18. If $p:7$ is an even integer and $q:\sqrt{2}$ is irrational the truth values of the compound propositions $p \vee q, p \wedge q$ and $p \vee q \to p \wedge q$ are respectively

(a) T, T, T (b) T, F, T (c) T, F, F (d) F, T, T

19. The converse of the proposition. If an integer is greater than 50, then it is greater than 20.

(a) If an integer is greater than 20, then it is greater than 50

(b) If an integer is not greater than 20, then it is not greater than 50

(c) If an integer is not greater than 50, then it is not greater than 20

(d) If an integer is greater than 50, then it is less than 20

20. The contra positive of the statement. If $x^2-5x+6=0$ then $x=2$ or $x=3$ is

(a) If $x^2-5x+6\neq 0$ then $x\neq 2$ or $x\neq 3$ (b) If $x=2$ or $x=3$ then $x^2-5x+6=0$

(c) If $x\neq 2$ or $x\neq 3$ then $x^2-5x+6\neq 0$ (d) If $x\neq 2$ and $x\neq 3$ then $x^2-5x+6\neq 0$

Answer Key:

1. c	2. c	3. b	4. d	5. b	6. d	7. b	8. c	9. c	10. a
11. d	12. a	13. c	14. b	15. b	16. b	17. a	18. c	19. b	20. d

Homework problems

1. The co-efficient of x^7 in the expansion of $\left(\frac{x^2}{2}-\frac{2}{x}\right)^9$ is

(a) -56 (b) 14 (c) -14 (d) none of these

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2. The co-efficient of x in the expansion of $\left(x^2+\frac{a}{x}\right)^5$ is

(a) $9a^2$ (b) $10a^3$ (c) $10a^2$ (d) $10a$

3. The terms containing x^3 in the expansion of $(x-2y)^7$ is

(a) 3*rd* (b) 4*th* (c) 5*th* (d) 6*th*

4. If the 21st and 22nd terms in the expansion of $(1+x)^{44}$, are equal, then $x=$

(a) $\frac{7}{8}$ (b) $\frac{8}{7}$ (c) $-\frac{7}{8}$ (d) $-\frac{8}{7}$

5. The term independent of y in the expansion of $\left(y^{\frac{1}{6}}-y^{-\frac{1}{3}}\right)^9$ is

(a) 84 (b) 8.4 (c) –0.84 (d) –84

6. The term independent of x in the expansion of $\left(x-\frac{3}{x^2}\right)^{18}$ is

(a) ${}^{18}C_6$ (b) ${}^{18}C_6 3^6$ (c) ${}^{18}C_{12}3^{-6}$ (d) 3^6

7. The middle term in the expansion of $\left(1+\frac{1}{x}\right)^{10}$ is

(a) $-{}^{10}C_5\frac{1}{x^5}$ (b) ${}^{10}C_6\frac{1}{x^6}$ (c) ${}^{10}C_5\frac{1}{x^5}$ (d) $-{}^{10}C_6\frac{1}{x^6}$

8. The coefficients of $(2r+4)th$ and $(r-2)th$ terms in $(x+1)^{18}$ are equal. Then the value of r is

(a) 3 (b) 5 (c) 6 (d) 8

9. If ${}^nC_{12}={}^nC_6$, then ${}^nC_2=$

(a) 72 (b) 153 (c) 306 (d) 2556

10. The co-efficient of x^{53} in $\sum_{r=0}^{100}{}^{100}C_r\,(x-3)^{100-r}\,2^r$ is

(a) ${}^{100}C_{51}$ (b) ${}^{100}C_{52}$ (c) $-{}^{100}C_{53}$ (d) ${}^{100}C_{54}$

11. The co-efficient of x^{-9} in the expansion of $\left(\frac{x^2}{2}-\frac{2}{x}\right)^9$ is

(a) 512 (b) –512 (c) 521 (d) 251

12. The co-efficient of x^5 in $(1+x^2)^5(1+x)^4$ is

(a) 20 (b) 30 (c) 60 (d) 55

13. The co-efficient of $\frac{1}{x}$ in the expansion of $\left(\frac{1}{x}+1\right)^n(1+x)^n$ is

(a) ${}^{2n}C_n$ (b) ${}^{2n}C_{n-1}$ (c) ${}^{2n}C_1$ (d) ${}^{n}C_{n-1}$

14. The value of ${}^{14}C_1+{}^{14}C_3+{}^{n}C_5+.....+{}^{14}C_{11}$ is

(a) $2^{14}-1$ (b) $2^{14}-14$ (c) 2^{12} (d) $2^{13}-14$

15. The value of ${}^{13}C_2+{}^{13}C_3+{}^{13}C_4+.....+{}^{13}C_{13}$ is

(a) $2^{13}-13$ (b) $2^{13}-14$

(c) an odd number $\neq 2^{13}-12$ (d) an even number $\neq 2^{13}-14$

16. The coefficient of x^{99} in $(x+1)(x+3)(x+5)....(x+199)$ is

(a) $1+2+3+....+99$ (b) $1+3+5+....+199$

(c) 1.3.5.....199 (d) none of these

17. If $(1+x)^n=C_0+C_1x+C_2x^2+...+C_nx^n$ then the sum of the series $C_0+{}^3C_1+{}^5C_2+.....+(2n+1)C_n$ will be

(a) $n.2^n$ (b) $(n+1)2^n$ (c) $n.2^n+1$ (d) 0

18. If $(1+x)^n=C_0+C_1x+C_2x^2+...+C_nx^n$ then $C_0C_r+C_1C_{r+1}+....+C_{n-r}C_n=$

(a) $\frac{(2n)!}{(n-r)!(n+r)}$ (b) $\frac{(2n)!}{(n+r)!(n-r)}$ (c) $\frac{(2n)!}{(n-r)!(n+r)!}$ (d) $\frac{(2n)!}{(n^2-r^2)!}$

19. The total number of terms in the expansion of $(x+y)^{100}+(x-y)^{100}$ after simplification is

(a) 50 (b) 51 (c) 202 (d) none of these

Subject	Topic	Lecture No.
Mathematics	Binomial Theorem, Mathematical logic	M-02

20. If $C_0, C_1, ..., C_n$ denote the binomial coefficients in the expansion of $(1+x)^n$, and the value of $C_0 + 2C_1 + 3C_2 + ... + (n+1)C_n$ is 576, then $n =$

(a) 7 (b) 8 (c) 9 (d) 10

21. The number of terms in the expansion of $(2x + y + 3z)^{10} =$

(a) 64 (b) 65 (c) 66 (d) none of these

22. Negation of the conditional : "If it rains, I shall go to school" is

(a) It rains and I shall go to school

(b) It rains and I shall not to go to school

(c) It does not rain and I shall go to school

(d) None of these

23. $\sim(p \wedge q)$ is equal to

(a) $\sim p \vee \sim q$ (b) $\sim p \wedge \sim q$ (c) $\sim p \wedge q$ (d) $p \wedge \sim q$

24. $\sim[p \vee (\sim q)]$ is equal to

(a) $\sim p \vee q$ (b) $(\sim p) \wedge q$ (c) $\sim p \vee \sim p$ (d) $\sim p \wedge \sim q$

25. $\sim(p \leftrightarrow q)$ is

(a) $\sim p \wedge \sim q$ (b) $\sim p \vee \sim q$ (c) $(p \wedge \sim q) \vee (\sim p \wedge q)$ (d) none of these

26. If $p \rightarrow (q \vee r)$ is false, then the truth values of p, q, r are respectively

(a) T, F, F (b) F, F, F (c) F, T, T (d) T, T, F

27. Which of the following is not a proposition

(a) 3 is a prime (b) $\sqrt{2}$ is irrational

(c) Mathematics is interesting (d) 5 is an even integer

28. The contrapositive of the inverse of $p \rightarrow \sim q$ is

(a) $\sim q \rightarrow p$ (b) $p \rightarrow q$ (c) $\sim q \rightarrow \sim p$ (d) $\sim p \rightarrow \sim q$

29. Let p : the child drinks milk

q : It stops crying Then $p \rightarrow q$ denotes

(a) The child drinks milk and it stops crying

(b) If the child drinks milk, then it stops crying

(c) The child drinks milk or it stops crying

(d) If and only if the child drinks milk, it stops crying

30. The contrapositive of $p \to (\neg q \to r)$ is

(a) $(\neg q \wedge r) \to \neg p$ (b) $(q \wedge \neg r) \to \neg p$ (c) $p \to \neg \vee q$ (d) $p \wedge (q \vee r)$

31. $\neg (p \to q) \leftrightarrow \neg p \vee \neg q$ is

(a) a tautology (b) a contradiction

(c) neither a tautology nor a contradiction (d) cannot come to any conclusion

32. The proposition $p \to \neg (p \wedge \neg q)$ is

(a) contradiction (b) a tautology

(c) either (a) or (b) (d) neither (a) nor (b)

33. $(p \wedge \neg q) \wedge (\neg p \wedge q)$ is

(a) a tautology (b) a contradiction

(c) both a tautology and a contradiction (d) neither a tautology nor a contradiction

Home Work Answers :

1. d	2. b	3. c	4. c	5. d	6. b	7. c	8. c	9. b	10.c
11.b	12.c	13.b	14.d	15.b	16.b	17.b	18.c	19.b	20.a
21.c	22.b	23.a	24.b	25.c	26.a	27.c	28.a	29.b	30.a
31.c	32.d	33.b							