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Mathematics	Sets, Relations, Functions	M-03

## Sets relation and functions

**Set:** A set is a well-defined collection of objects. The objects comprising the set are called its elements.

**Types:** (1) Roaster method:  $A = \{1, 2, 3, 4, \dots, n\}$

(2) Set builder form:  $A = \{x : x \text{ is a natural number}\}$   $A =$

Ex:  $A = \{a, e, i, o, u\}$ .

If  $x$  is any element of set  $A$  then it is denoted as  $x \in A$  and if  $x$  is not a element of set  $A$  then it is denoted by  $x \notin A$ .

If the number of elements in a set is finite then it is called finite set, otherwise it is said to be infinite set.

A set with single element is called singleton set and a set which contains no elements is called an empty set or null set and is denoted by  $\phi$ .

Ex:  $N = \{x : x \text{ is a natural number}\}$  is an infinite set

$C = \{2\}$  is a singleton set

$D = \{x : x^2 = 9 \& x \text{ is even}\}$  is a null set.

**Subsets:** If every element of set  $A$  is also an element of a set  $B$  then  $A$  is called a subset of  $B$  and it is denoted by  $A \subset B$  or  $B \supset A$ .

Ex:  $A = \{1, 2, 3, 4\}, B = \{1, 2, 3, 4, 5, 6\}$   $\therefore A \subset B$ .

A set ' $A$ ' is said to be a proper subset of ' $B$ ' if there exists an element of  $B$  which is not an element of  $A$ . That is  $A$  is a proper subset of  $B$  if  $A \subset B$  and  $A \neq B$ .

Ex: If  $A = \{1, 3, 5\}, B = \{1, 3, 5, 7\}$  then  $A$  is proper subset of  $B$ .

Two sets  $A$  and  $B$  are said to be equal iff  $A \subset B$  and  $B \subset A$ .

**Power set** ( $P(A)$ ):- The set of all subsets of a set  $A$  is called the power set of  $A$  & its denoted by  $P(A)$ .

Ex:  $A = \{1, 3, 5\}$  then  $P(A) = \{\phi, \{1\}, \{3\}, \{5\}, \{1, 3\}, \{1, 5\}, \{3, 5\}, \{1, 3, 5\}\}$

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\* If a set has ' $n$ ' elements then its power set  $P(A)$  has  $2^n$  elements.

**Note:** Non-empty subsets =  $2^n - 1$ .

**Cardinality of a set:** If  $A$  is finite set then the cardinality of  $A$  is the total number of elements that comprise the set and is denoted by  $n(A)$ .

Ex:  $A = \{a, b\}$  then  $n(A) = 2$

**Universal sets:** Every set is a subset of some fixed set. This fixed set is called the universal set. It is denoted by  $U$ .

**Union of sets:** The union of two sets  $A$  and  $B$  denoted by  $A \cup B$  is the set of elements which belong to  $A$  or  $B$  or both.

That is  $A \cup B = \{x: x \in A \text{ or } x \in B\}$

**Properties:**

- $A \cup B = A \cup B$
- $A \cup B = B \cup A$
- $A \cup (B \cap C) = (A \cup B) \cap C$
- $A \subset A \cup B$  and  $B \subset A \cup B$

**Intersection of sets:** The intersection of two sets  $A$  and  $B$  denoted by  $A \cap B$  is the set of elements which belong to both  $A$  and  $B$ .

That is  $A \cap B = \{x: x \in A \text{ and } x \in B\}$ .

**Properties:**

- $A \cap A = A$
- $A \cap B = B \cap A$
- $A \cap (B \cap C) = (A \cap B) \cap C$
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

If  $A \cap B = \phi$  then  $A$  and  $B$  are said to be disjoint sets.

**Difference of sets:** The difference of two sets  $A$  and  $B$  denoted by  $A - B$  is the set of elements of  $A$  which are not the elements of  $B$  that is,  $A - B = \{x: x \in A, x \notin B\}$ .

- $A - B \subset A$

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2.  $A - B \neq B - A$
3.  $A - B, A \cap B, B - A$  are mutually disjoint sets.

**Complement of a set:** The complement of a set  $A$  with respect to the universal set  $U$  is defined as  $U - A$  and is denoted by  $A'$  or  $A^c$ . That is  $A' = \{x : x \in U, x \notin A\}$

(a)  $(A')' = A$       (b)  $\phi' = U$       (c)  $U' = \phi$ .

**Demorgan's Laws :** For any 3 sets  $A, B, C$

1.  $(A \cup B)' = A' \cap B'$
2.  $(A \cap B)' = A' \cup B'$
3.  $A - (B \cup C) = (A - B) \cap (A - C)$
4.  $A - (B \cap C) = (A - B) \cup (A - C)$ .

**Cartesian product of two sets:** Let  $A$  and  $B$  be two sets. Then the Cartesian product of  $A$  and  $B$  is defined as the set of all ordered pairs  $(x, y)$  where  $x \in A, y \in B$  and is denoted by  $A \times B$ . Thus  $A \times B = \{(x, y) : x \in A \text{ and } y \in B\}$ .

\* If  $A$  contains ' $n$ ' elements and  $B$  contains ' $m$ ' elements then  $A \times B$  contains  $mn$  ordered pairs.

**Some important results:**

(1)  $A$  &  $B$  are disjoint sets.  $n(A \cup B) = n(A) + n(B)$

(2)  $A$  &  $B$  are not disjoint sets

(i)  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii)  $n(A) = n(A - B) + n(A \cap B)$

(iii)  $n(B - A) = n(B) - n(A \cap B)$

(iv)  $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

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**Relations:** Let  $A$  &  $B$  be two non empty sets. Then a relation  $R$  from  $A$  to  $B$  is a subset of  $A \times B$  containing the ordered pairs  $(a,b) \in A \times B$  such that some relation exists between  $a$  &  $b$ .

If  $(a,b) \in R$  then we say that "  $a$  is  $R$  related to  $b$  " and is written as  $aRb$ .

Ex:  $A = \{2,3,5\}$   $B = \{4,6,9\}$  then

$A \times B = \{(2,4), (2,6), (2,9), (3,4), (3,6), (3,9), (5,4), (5,6), (5,9)\}$  define  $R$  by  $R = \{(a,b) \in A \times B : a \text{ divides } b\}$ . Then  
 $R = \{(2,4), (2,6), (3,6), (3,9)\}$ .

**Domain and Range of a Relation:** Let  $R$  be a relation from  $A$  to  $B$ , then the domain of  $R$  is the set of all first coordinates of the ordered pairs of  $R$  and the range of  $R$  is the set of second co-ordinates of the pairs of  $R$  that is

$$\text{Domain of } R = \{a \in A : (a,b) \in R\}$$

$$\text{Range of } R = \{b \in B : (a,b) \in R\}$$

**Inverse of a relations:** Let  $R$  be a relation from set  $A$  to a set  $B$  then the inverse relation of  $R$  denoted by  $R^{-1}$  is the relation from  $B$  to  $A$  defined by  $R^{-1} = \{(b,a) : (a,b) \in R\}$

## Types of Relations

1. **Reflexive relation:** A relation  $R$  in a set  $A$  is said to be reflexive if for every  $a \in A, (a,a) \in A$ . Thus  $R$  is reflexive if we have  $aRa$  for every  $a \in A$ .

Ex: " $x = y$ " is reflexive since every number is equal to itself in the set of natural numbers.

2. **Symmetric Relation:** A relation  $R$  in a set  $A$  is said to be symmetric if  $(a,b) \in R$  implies  $(b,a) \in R$ . That is  $R$  is said to be symmetric if  $aRb \Rightarrow bRa$ .

Ex: In a set of natural numbers " $x - y$  is divisible by 5" is a symmetric relation since if  $x - y$  is divisible by 5 then  $y - x$  is also divisible by 5.

3. **Anti-symmetric Relation:** A relation  $R$  in a set 'A' is said to be an anti-symmetric relation if  $(a,b) \in R$  and  $(b,a) \in R$  implies  $a = b$ .

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Ex:  $A = \{1,2,3,4\}$  and  $R$  be a relation defined by  $R = \{(1,2)(2,1)(3,1)(4,2)(4,4)\}$  then  $R$  is not anti-symmetric relation in  $A$  since  $(1,2) \in R$  and  $(2,1) \in R$  but  $1 \neq 2$

**4. Transitive relation:** A relation  $R$  in a set ' $A$ ' is said to be transitive if  $(a,b) \in R$  and  $(b,c) \in R$  implies  $(a,c) \in R$

Ex: " $x < y$ " is transitive since  $x < y$  and  $y < z$  implies  $x < z$ .

**5. Equivalence relation:** A relation  $R$  in a set ' $A$ ' is said to be an equivalence relation iff,

- (i)  $R$  is reflexive                      (ii)  $R$  is symmetric                      (iii)  $R$  is transitive

### Functions:

Let  $A$  and  $B$  be two non-empty sets. A function or mapping ' $f$ ' from  $A$  to  $B$  is a relation which associates every element of  $A$  with a unique element of  $B$  and is denoted by  $f: A \rightarrow B$ .

If  $f: A \rightarrow B$  is a function then  $A$  is called the Domain and  $B$  is called co-domain of  $f$ . If  $x \in A$  is associated with a unique element  $y \in B$  by function  $f$ , then  $y$  is called image of  $x$  under  $f$  and is denoted by  $y = f(x)$ . Also  $x$  is called the pre-image of  $y$  under  $f$ .

The range of  $f$  is the set of those elements of  $B$  which appear as the image of at least one element of  $A$  and is denoted by  $f(A)$ .

Thus,  $f(A) = \{f(x) \in B : x \in A\}$  clearly  $f(A) \subset B$ .

### Types of Functions:

**1. One –one function:** A function  $f: A \rightarrow B$  is said to be one-one if different elements of  $A$  have different images in  $B$ . It is also called injective function.

**2. Onto function:** A function  $f: A \rightarrow B$  is said to be onto if for every  $y \in B$  there exists at least one element  $x \in A$  such that  $f(x) = y$ . These if  $f$  is onto then  $f(A) = B$ . It is also called surjective function.

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- 3. One-to-one function:** A function  $f: A \rightarrow B$  is said to be one-to-one (or) bijective if it is both one-one and onto .
- 4. Into function:**  $f: A \rightarrow B$  is an into function if the range of  $f$  is a proper subset of  $B$
- 5. Many-one functions:**  $f: A \rightarrow B$  is said to be a many -one function if more than one element of  $A$  is mapped to an element of  $B$  .
- 6. Identity function:**  $f: A \rightarrow B$  is said to be an identity function if  $f(x) = x, \forall x \in A$  Here  $B = A$  and  $f$  becomes I
- 7. Constant function:**  $f: A \rightarrow B$  is said to be constant function if  $f(x) = K, \forall x \in A$  where  $K$  is a fixed element of  $B$  .
- 8. Odd function:** A function  $f(x)$  is called an odd function if  $f(-x) = -f(x)$ .
- 9. Even function:** A function  $f(x)$  is called an even function if  $f(-x) = f(x)$ .
- 10. Periodic function** If  $f(x)$  is the given function and if  $f(x) = f(x+T) = f(x+2T) = \dots = f(x+nT)$ , where  $T$  is the period, then  $f(x)$  is called a periodic function.
- 11. Inverse function:** If  $f: A \rightarrow B$  is a bijective function then the inverse of  $f$  denoted by  $f^{-1}: B \rightarrow A$  is defined by  $f^{-1} = \{(y, x): (x, y) \in f\}$ .

**Composite function:** If  $f: A \rightarrow B$  and  $g: B \rightarrow C$  are two functions then the composite function of  $f$  and  $g$  denoted by  $g \circ f$  is a function from  $A$  to  $C$  defined by,  $(g \circ f)(x) = g\{f(x)\}$  for every  $x \in A$

Note:  $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ .

### Class work Problems:

- Let  $\cup = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ,  $A = \{1, 2, 5\}$ ,  $B = \{6, 7\}$  then  $A \cap B'$  is
  - $B'$
  - $A$
  - $A'$
  - $B$
- $A = \{2, 3, 4, 8, 10\}$ ,  $B = \{3, 4, 5, 10, 12\}$ ,  $C = \{4, 5, 6, 12, 14\}$ , then  $(A \cup B) \cap (A \cup C)$  is equal to

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- (a)  $\{2,3,4,5,8,10,12\}$  (b)  $\{2,4,8,10,12\}$  (c)  $\{3,8,10,12\}$  (d)  $\{2,8,10\}$
3. If  $A$  and  $B$  are disjoint, then  $n(A \cup B)$  is equal to  
 (a)  $n(A)$  (b)  $n(B)$  (c)  $n(A)+n(B)$  (d)  $n(A) \cdot n(B)$
4. If  $A = \{2,4,5\}, B = \{7,8,9\}$ , then  $n(A \times B)$  is equal to  
 (a) 6 (b) 9 (c) 3 (d) 0
5. If a set  $A$  has  $n$  elements, then the total number of subsets of  $A$  is  
 (a)  $n$  (b)  $n^2$  (c)  $2^n$  (d)  $2n$
6. Sets  $A$  and  $B$  have 3 and 6 elements respectively. What can be the minimum number of elements in  $A \cup B$ ?  
 (a) 3 (b) 6 (c) 9 (d) 18
7. If  $A = \{1,2,3,4,5\}$ , then the number of proper subsets of  $A$  is  
 (a) 120 (b) 30 (c) 31 (d) 32
8. Two finite sets have  $m$  and  $n$  elements. The total number of subsets of the first set is 48 more than the total number of subsets of the second set. The values of  $m$  and  $n$  are  
 (a) 7, 6 (b) 6, 3 (c) 6, 4 (d) 7, 4
9. In a college of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. The number of newspapers is  
 (a) at least 30 (b) at most 20 (c) exactly 25 (d) none of these
10. If  $A = \{x : x^2 - 5x + 6 = 0\}, B = \{2,4\}, C = \{4,5\}$ , then  $A \times (B \cap C)$  is  
 (a)  $\{(2,4), (3,4)\}$  (b)  $\{(4,2), (4,3)\}$   
 (c)  $\{(2,4), (3,4), (4,4)\}$  (d)  $\{(2,2), (3,3), (4,4), (5,5)\}$

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11. If two sets  $A$  and  $B$  are having 99 elements in common, then the number of elements common to each of the sets  $A \times B$  and  $B \times A$  are
- (a)  $2^{99}$                       (b)  $99^2$                       (c) 100                      (d) 18
12.  $R$  be the relation on the set  $N$  of natural numbers, defined by  $xRy$  if and only if  $x + 2y = 8$ . The domain of  $R$  is
- (a)  $\{2, 4, 7\}$                       (b)  $\{1, 2, 4\}$                       (c)  $\{2, 4, 6\}$                       (d)  $\{2, 6, 8\}$
13. Let  $A = \{a, b, c\}$  and  $R = \{(a, a), (b, b), (a, b), (b, a), (b, c)\}$  be a relation on  $A$ , then  $R$  is
- (a) reflexive                      (b) symmetric                      (c) transitive                      (d) none of these
14. Let  $A = \{p, q, r\}$  which of the following is an equivalence relation on  $A$ ?
- (a)  $R_1 = \{(p, q), (q, r), (p, r), (p, p)\}$                       (b)  $R_2 = \{(r, p), (q, p), (r, r), (q, q)\}$
- (c)  $R_3 = \{(p, p), (q, q), (r, r), (p, q)\}$                       (d)  $R_4 = \{(p, p), (q, q), (r, r)\}$
15. Let  $W$  denote the words in an English dictionary. Define relation  $R$  by  $R = \{(x, y) \in W \times W : \text{the word } x \text{ and } y \text{ have at least one letter in common}\}$  Then  $R$  is
- (a) reflexive, not symmetric and transitive                      (b) not reflexive, symmetric and transitive
- (c) reflexive, symmetric and not transitive                      (d) reflexive, symmetric and transitive
16. If  $A = \{2, 3, 4, 5\}$ , then which of the following relations is a functions from  $A$  to itself
- (a)  $f_1 = \{(x, y) : y = x + 1\}$                       (b)  $f_2 = \{(x, y) : x + y > 6\}$
- (c)  $f_3 = \{(x, y) : x > y\}$                       (d)  $f_4 = \{(x, y) : x + y = 7\}$
17. If  $f : N \times N \rightarrow N$  is such that  $f(m, n) = m + n$ , for all  $n \in N$ , where  $N$  is the set of all natural numbers, then which of the following is true?
- (a)  $f$  is one-one but not onto                      (b)  $f$  is neither one-one nor onto



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(c)  $f$  is one-one and onto (d)  $f$  is onto but not one-one

18. The domain of the function

$$f(x) = \sqrt{(2-2x-x^2)} \text{ is}$$

(a)  $-1 \leq x \leq \sqrt{3}$  (b)  $-1 - \sqrt{3} \leq x \leq -1 + \sqrt{3}$  (c)  $-2 \leq x \leq 2$  (d) none of these

19. If  $f(x) = 1 + x^4$ , then  $f(x) \cdot f\left(\frac{1}{x}\right) =$

(a)  $f(x) + f\left(\frac{1}{x}\right)$  (b)  $f(x) - f\left(\frac{1}{x}\right)$  (c)  $f(x) + f\left(\frac{1}{x}\right)$  (d) none of these

20. The number of objective functions from the set  $A$  to itself, if  $A$  contains 108 elements is

(a) 108 (b)  $(108)!$  (c)  $(108)^2$  (d)  $2^{108}$

21. The domain of the function

$$f(x) = \sqrt{x-1} + \sqrt{6-x} \text{ is}$$

(a)  $[1, \infty)$  (b)  $(-\infty, 6)$  (c)  $[1, 6]$  (d)  $(-\infty, 6]$

22. The range of the function  $f(x) = \frac{x^2 - x}{x^2 + 2x}$  is

(a)  $R - \left\{-\frac{1}{2}, 1\right\}$  (b)  $R$  (c)  $R - \{1\}$  (d) none of these

23. Domain of  $\sin^{-1}\left(\frac{2x+1}{3}\right)$  is

(a)  $(-2, 1)$  (b)  $[-2, 1]$  (c)  $R$  (d)  $(-2, 0)$

24. The range of the function  $f(x) = {}^{7-x}P_{x-3}$  is

(a)  $\{1, 2, 3, 4\}$  (b)  $\{1, 2, 3, 4, 5, 6\}$  (c)  $\{1, 2, 3\}$  (d)  $\{1, 2, 3, 4\}$

25. The domain of  $\sqrt{(x-2)(3-x)}$  is

(a)  $(2, 3)$  (b)  $(2, 3]$  (c)  $[2, 3]$  (d) none of these

26. Let  $f(x) = \frac{x-1}{x+1}$ , then  $f(f(x))$  is

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- (a)  $\frac{1}{x}$                       (b)  $-\frac{1}{x}$                       (c)  $\frac{1}{x+1}$                       (d)  $\frac{1}{x-1}$

27. Let  $f(x) = \frac{x-1}{x+1}$ ,  $x \neq -1$ , then  $f^{-1}(x)$  is

- (a)  $\frac{x+1}{x-1}$                       (b)  $\frac{1+x}{1-x}$                       (c)  $\frac{2}{1+x}$                       (d)  $\frac{1}{x-1}$

28. If  $f(x) = 1 - \frac{1}{x}$ , then  $f\left(f\left(\frac{1}{x}\right)\right)$  is

- (a)  $\frac{1}{x}$                       (b)  $\frac{1}{1+x}$                       (c)  $\frac{x}{x-1}$                       (d)  $\frac{1}{x-1}$

29. If  $f(x) = \log\left(\frac{1+x}{1-x}\right)$  and  $g(x) = \frac{3x+x^3}{1+3x^2}$ , then  $f(g(x))$  is equal to

- (a)  $-f(x)$                       (b)  $3f(x)$                       (c)  $(f(x))^3$                       (d)  $f(3x)$

30. If  $f: R \rightarrow R$  and  $g: R \rightarrow R$  defined by  $f(x) = 2x+3$  and  $g(x) = x^2+7$ , then the values of  $x$  such that  $g(f(x)) = 8$  are

- (a) 1, 2                      (b) -1, 2                      (c) -1, -2                      (d) 1, -2

31. If  $f(x) = (25-x^4)^{1/4}$  for  $0 < x < \sqrt{5}$ , then  $f\left(f\left(\frac{1}{2}\right)\right) =$

- (a)  $2^{-4}$                       (b)  $2^{-3}$                       (c)  $2^{-2}$                       (d)  $2^{-1}$

32. If  $g(f(x)) = |\sin x|$  and  $f(g(x)) = (\sin \sqrt{x})^2$  then

- (a)  $f(x) = \sin^2 x, g(x) = \sqrt{x}$                       (b)  $f(x) = \sin x, g(x) = |x|$   
(c)  $f(x) = x^2, g(x) = \sin \sqrt{x}$                       (d)  $f$  and  $g$  cannot be determined

33. If  $f: [1, \infty) \rightarrow [2, \infty)$  is given by  $f(x) = x + \frac{1}{x}$ , then  $f^{-1}(x) =$

- (a)  $\frac{x + \sqrt{x^2 - 4}}{2}$                       (b)  $\frac{x}{1+x^2}$                       (c)  $\frac{x - \sqrt{x^2 - 4}}{2}$                       (d)  $1 + \sqrt{x^2 - 4}$

34. Let  $f: [0,1] \rightarrow [0,1]$  and  $g: [0,1]$  will be two functions defined by  $f(x) = \frac{1-x}{1+x}$  and

$g(x) = 4x(1-x)$ , then  $f \circ g(x) =$

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- (a)  $\frac{8x(1-x)}{(1+x)^2}$       (b)  $\frac{4(1-x)}{1+x}$       (c)  $\frac{1-4x+4x^2}{1+4x-4x^2}$       (d) none of these

35. If  $f: (R) \rightarrow R$  and  $g: R \rightarrow R$  defined by  $f(x) = 2x + 3$  and  $g(x) = x^2 + 7$ , then the value of  $x$  for which  $f(g(x)) = 25$  are

- (a)  $\pm 1$       (b)  $\pm 2$       (c)  $\pm 3$       (d)  $\pm 4$

### Class work Answer Key

1. b	2. a	3. c	4. b	5. c	6. b	7. c	8. c	9. c	10.a
11.b	12.c	13.d	14.d	15.c	16.d	17.b	18.b	19.a	20.b
21.c	22.a	23.b	24.c	25.c	26.b	27.b	28.c	29.b	30.c
31.d	32.a	33.a	34.c	35.b					

### Homework Problems

- Let  $A = \{a, b, c\}$ ,  $B = \{b, c, d\}$ ,  $C = \{a, b, d, e\}$ , then  $A \cap (B \cap C)$  is
 

(a)  $\{a, b, c\}$       (b)  $\{b, c, d\}$       (c)  $\{a, b, d, e\}$       (d)  $\{e\}$
- If  $A$  and  $B$  are not disjoint, then  $n(A \cup B)$  is equal to
 

(a)  $n(A) + n(B)$       (b)  $n(A) + n(B) - n(A \cap B)$   
 (c)  $n(A) + n(B) + n(A \cap B)$       (d)  $n(A) \cdot n(B)$
- If  $n(A) = 3$  and  $n(B) = 6$  and  $A \subseteq B$ , then the number of elements in  $A \cup B$  is equal to
 

(a) 3      (b) 9      (c) 6      (d) none of these
- If  $A = \{0, 1\}$ , and  $B = \{1, 0\}$ , then  $A \times B$  is equal to:
 

(a)  $\{0, 1, 1, 0\}$       (b)  $\{(0, 1), (1, 0)\}$   
 (c)  $\{0, 0\}$       (d)  $\{(0, 1), (0, 0), (1, 1), (1, 0)\}$
- The number of non-empty subsets of the set  $\{1, 2, 3, 4\}$  is

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- (a) 15                      (b) 14                      (c) 16                      (d) 17
6. If the set  $A$  has  $p$  elements,  $B$  has  $q$  elements, then the number of elements in  $A \times B$  is
- (a)  $p + q$                       (b)  $p + q + 1$                       (c)  $pq$                       (d)  $p^2$
7. If  $A = \{a, b, c\}$ ,  $B = \{b, c, d\}$  and  $C = \{a, b, c\}$ , then  $(A - B) \times (B \cap C) =$
- (a)  $\{(a, b), (c, d)\}$                       (b)  $\{(a, c), (a, d)\}$   
(c)  $\{(a, c), (a, d), (b, d)\}$                       (d)  $\{(c, a), (d, a)\}$
8. In a city 20 percent of the population travels by car, 50 percent travels by bus and 10 percent travels by both car and bus. Then persons travelling by car or bus is
- (a) 80 percent                      (b) 40 percent                      (c) 60 percent                      (d) 70 percent
9. A class has 175 students. The following data shows the number of students obtaining one or more subjects. Mathematics 100; Physics 70; Chemistry 40; Mathematics and Physics 30; Mathematics and Chemistry 28; Physics and Chemistry 18. How many students have offered Mathematics alone?
- (a) 35                      (b) 48                      (c) 60                      (d) 22
10. Let  $X = \{1, 2, 3\}$  and  $R = \{(1, 1), (2, 2), (3, 3), (2, 3)\}$  be a relation on  $X$ . Then which one is not true
- (a)  $R$  is reflexive    (b)  $R$  is transitive    (c)  $R$  is antisymmetric    (d)  $R$  is symmetric
11. Let  $A = \{p, q, r, s\}$  and  $B = \{1, 2, 3\}$  which of the following relations from  $A$  and  $B$  is not a function
- (a)  $R_1 = \{(p, 1), (q, 2), (r, 1), (s, 2)\}$                       (b)  $R_2 = \{(p, 1), (q, 1), (r, 1), (s, 1)\}$   
(c)  $R_3 = \{(p, 1), (p, 2), (r, 2), (s, 3)\}$                       (d)  $R_4 = \{(p, 2), (q, 3), (r, 2), (s, 2)\}$
12. In the set  $X = \{a, b, c, d\}$  which of the following relations is function?
- (a)  $R_1 = \{(b, a), (a, b), (c, d), (a, c)\}$                       (b)  $R_2 = \{(a, d), (d, c), (b, b), (c, c)\}$

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(c)  $R_3 = \{(a,b), (b,c), (c,d), (b,d)\}$                       (d)  $R_4 = \{(a,a), (b,b), (c,c), (c,d)\}$

13. The relation  $R = \{(1,1), (2,2), (3,3)\}$  on the set  $\{1,2,3\}$  is

- (a) symmetric only    (b) reflexive only  
(c) an equivalence relation                                      (d) transitive only

14. If  $R$  be the relation on the set  $R$  of all real numbers defined by  $aRb$  if and only if  $|a-b| \leq 1$ . Then  $R$  is

- (a) reflexive and symmetric                                      (b) symmetric only  
(c) transitive only    (d) anti symmetric

15. Let  $A = \{1,2,3\}$  and  $B = \{2,3,4\}$ , then which of the following relation from  $A$  to  $B$  is a function from  $A$  into  $B$

- (a)  $\{(2,2), (1,3), (2,4), (3,2)\}$                                       (b)  $\{(1,4), (2,4), (3,4)\}$   
(c)  $\{(2,2), (3,4)\}$     (d)  $\{(1,2), (2,3), (3,4), (3,3)\}$

16. Let  $A = \{1,2,3,4\}$ ,  $B = \{1,2\}$ . Then the number of onto functions from  $A$  onto  $B$  is

- (a) 14                                      (b) 16                                      (c) 12                                      (d) 8

17. If  $X = \{1,2,3,4\}$ , then one-one onto mappings  $f: X \rightarrow X$  such that  $f(1)=1, f(2) \neq 2$  and  $f(4) \neq 4$  are given by

- (a)  $\{(1,1), (2,3), (3,4), (4,2)\}$                                       (b)  $\{(1,2), (2,4), (3,3), (4,2)\}$   
(c)  $\{(1,2), (2,4), (3,2), (4,3)\}$                                       (d) none of these

18. The domain of the function  $f(x) = \sqrt{\log_{10} x^2}$  is

- (a)  $x \geq 0$                                       (b)  $|x| \geq 1$                                       (c)  $|x| \leq 1$                                       (d)  $|x| \geq 4$

19. Domain of the definition of  $f(x) = \frac{\log_2(x+3)}{x^2+3x+2}$  is

- (a)  $R - \{-1, -2\}$                                       (b)  $(-2, \infty)$                                       (c)  $R - \{-1, -2, -3\}$                                       (d)  $(-3, \infty) - \{-1, -2\}$

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20. Range of the function  $f(x) = \frac{x^2 + x + 2}{x^2 + x + 1}$  where  $x$  is a real number is

- (a) (3,5)                      (b) [1,3]                      (c)  $\left[1, \frac{7}{5}\right]$                       (d)  $\left(1, \frac{7}{3}\right)$

21. The domain of the function  $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$  is

- (a) (1,2)                      (b) [2,3]                      (c) [2,3]                      (d) [1,2]

22. The number of bijective functions (one-one onto) from set A to itself when A contains 106 elements is

- (a) 106                      (b)  $106^3$                       (c)  $106!$                       (d)  $2^{106}$

23. Let A, B be two sets each with 10 elements. Then the number of all possible bijections from A to B is

- (a) 20                      (b)  $10!$                       (c) 100                      (d) none of these

24. Domain of  $\sqrt{4-x^2}$  is

- (a) (-2,2)                      (b) (-3,2]                      (c) [-2,2]                      (d) none of these

25. Domain of  $\sqrt{x^2-16}$  is

- (a) [-4,4]                      (b)  $(-\infty, 4) \cup (4, \infty)$                       (c)  $(-\infty, -4] \cup [4, \infty)$                       (d) none of these

26. The domain of  $\sqrt{(x-3)(x-5)}$  is

- (a)  $(-\infty, 3) \cup (5, \infty)$                       (b)  $(-\infty, 3] \cup [5, \infty)$                       (c)  $(-\infty, 3] \cup (5, \infty)$                       (d) none of these

27. The domain of  $\frac{1}{\sqrt{(x-4)(x-5)}}$  is

- (a)  $(-\infty, 4) \cup (5, \infty)$                       (b)  $(-\infty, 4] \cup [5, \infty)$                       (c)  $(-\infty, 4] \cup (5, \infty)$                       (d) none of these

28. If  $f(x) = \frac{x}{x-1}, x \neq 1$ , then  $f^{-1}(x)$  is

- (a)  $\frac{x-1}{x}$                       (b)  $\frac{x}{x-1}$                       (c)  $\frac{1-x}{x}$                       (d)  $\frac{1}{x-1}$

29. If  $f(x) = (a-x^n)^{1/n}$ , then  $f(f(x))$  equals:

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- (a)  $x$                       (b)  $a-x$                       (c)  $x^n$                       (d)  $x^{1/n}$

30. If  $f(x) = 2x^2 + x + 1$  and  $g(x) = 3x + 1$ , then  $(fog)(2) =$

- (a) 34                      (b) 33                      (c) 106                      (d) 105

31. If  $x \neq 1$  and  $f(x) = \frac{x+1}{x-1}$  is a real function then  $f(f(f(2)))$  is

- (a) 2                      (b) 4                      (c) 8                      (d) none of these

32. If  $f(x) = 3x - 5$ , then  $f^{-1}(x) =$

- (a)  $\frac{1}{3x-5}$                       (b)  $\frac{x+5}{3}$

- (c) does not exist because  $f$  is not one-one                      (d) does not exist because  $f$  is not onto

**Homework Answer Key**

1. a	2. b	3. c	4. d	5. a	6. c	7. b	8. c	9. c	10. d
11. c	12. b	13. c	14. a	15. b	16. a	17. a	18. b	19. d	20. d
21. b	22. c	23. b	24. c	25. c	26. b	27. a	28. b	29. a	30. c
31. a	32. b								