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Mathematics	Partial Fraction, Logarithm	M-01

Classwork Solutions-Partial fractions

1. Clearly $\frac{3(6)}{6+a}=2 \Rightarrow 9=6+a \quad \therefore a=3$

2. $\frac{3x+a}{(x-1)(x-2)}=\frac{A}{x-2}-\frac{10}{x-1}$

   $\therefore \frac{3+a}{1-2}=-10 \quad \Rightarrow 3+a=10 \quad \therefore a=7 \qquad \frac{6+a}{2-1}=A \quad \Rightarrow 13=A$

3. $\frac{x+1}{(x-1)(x-2)(x-3)}$

   $=\frac{A}{x-1}+\frac{B}{x-2}+\frac{C}{x-3} \qquad A=\frac{2}{(1-2)(1-3)}=1, \quad B=\frac{3}{1(-1)}=-3; \quad C=\frac{4}{2.1}=2$

4. $\frac{9}{(x-1)(x+2)^2}=\frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$

   $\therefore 9=A(x+2)^2+B(x-1)(x+2)+C(x-1).....(1)$

   Putting $x=1$, on both sides of (1), we get $9=A(3)^2 \quad \therefore A=1$

   Putting $x=-2,$ we get $9=C(-2-1) \quad \therefore C=-3$

   Compare constant term on both sides of (1), $9=4A-2B-C=4-2B+3$

   $\therefore 2=-2B \therefore B=-1 \quad \therefore$ required value $=\frac{1}{x-1}-\frac{1}{x+2}-\frac{3}{(x+2)^2}$

5. Since $\frac{1}{(1-ax)(1-bx)}=\frac{A}{1-ax}+\frac{B}{1-bx}$

   $\frac{1}{(1-ax)^2(1-bx)}=\frac{1}{(1-ax)}.\left[\frac{A}{1-ax}+\frac{B}{1-bx}\right]=\frac{A}{(1-ax)^2}+\frac{B}{(1-ax)(1-bx)}$

   $=\frac{A}{(1-ax)^2}+B\left[\frac{A}{1-ax}+\frac{B}{1-bx}\right]=\frac{A}{(1-ax)^2}+\frac{AB}{1-ax}+\frac{B^2}{1-bx}$

6. $3x+4=A(x+1)^2+B(x+1)(x-1)+C(x-1)$ Putting $x=1$ on both sides, we get

   $3+4=A(1+1)^2=4A \quad \therefore \quad A=\frac{7}{4}$

7. L.H.S. $=\frac{1-x+6x^2}{x(1-x^2)}=\frac{1-x+6x^2}{x(1-x)(1+x)} \qquad A=\frac{1-0+0}{(1-0)(1+0)}=1$

[By putting $x=0$ in $\frac{1-x+6x^2}{(1+x)(1+x)}$]

8. $a=\frac{1}{(1-2)(1-3)}=\frac{1}{2}$ by putting $x=1$ in $\frac{1}{(1-2x)(1-3x)}$

$b=\frac{1}{\left(1-\frac{1}{2}\right)\left(1-\frac{3}{2}\right)}=-4$ $\qquad$ $c=\frac{1}{\left(1-\frac{1}{3}\right)\left(1-\frac{2}{3}\right)}=\frac{9}{2}$

$\therefore \frac{a}{1}+\frac{b}{3}+\frac{c}{5}=\frac{1}{2}-\frac{4}{3}+\frac{9}{10}=\frac{15-40+27}{30}=\frac{2}{30}=\frac{1}{15}$

9. $Q=\frac{\text{co-efficiency of } x^3 \text{ in the numerator}}{\text{co-efficiency of } x^3 \text{ in the denominator}}=\frac{1\times2\times3}{(-2)\times(-3)\times(-4)}=-\frac{1}{4}.$

10. $\frac{2x^2+3x+4}{(x-1)(x^2+2)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+2}$

$\therefore 2x^2+3x+4=A(x^2+2)+(Bx+C)(x-1)$ putting $x=1,$ we get $9=A(3)\therefore A=3.$

$x^2)2=A+B=3+B\therefore B=-1,$ $\quad x)3=C-B=C+1\therefore C=2$

$\therefore$ required value $=\frac{3}{x-1}+\frac{-x+2}{x^2+2}=\frac{3}{x-1}-\frac{x-2}{x^2+2}$

11. Put $x^2=y$, $\quad \therefore \frac{y}{(y+a^2)(y+b^2)}=\frac{Ka^2}{y+a^2}-\frac{Kb^2}{y+b^2}$ $\quad \therefore \frac{-a^2}{b^2-a^2}=Ka^2$

$\therefore K=-\frac{1}{b^2-a^2}=\frac{1}{a^2-b^2}$

12. By short cut method

Remainder $=(-1)^{64}+(-1)^{27}+1=1-1+1=1$

13. Multiply both sides by x^3+x *i.e.* $x(x^2+1),$ we get $x^2+2x+1=A(x^2+1)+(Bx+C)x$

Putting $x=1$ on both sides, we get $1+2+1=2A+B+C$ *i.e.* $2A+B+C=4$

Compare constant term on both sides $1=A$

Now 2A+B+C=4 $\Rightarrow A+(A+B+C)=4\Rightarrow A+B+C=4-A=4-1=3$

14. $x^4+x^2+1=x^4+2x^2+1-x^2=(x^2+1)^2-x^2=(x^2+x+1)(x^2-x+1)$

$\therefore$ given fraction $=\frac{x^2+1}{(x^2+x+1)(x^2-x+1)}=\frac{Ax+B}{x^2+x+1}+\frac{Cx+D}{x^2-x+1}$

15. Clearly $\frac{3x^2-4x+5}{(x^2-x+1)^3}=\frac{Ax+B}{x^2-x+1}+\frac{Cx+D}{(x^2-x+1)^2}+\frac{Ex+F}{(x^2-x+1)^3}$

$\therefore$ there are three partial fractions.

16. Let $\frac{x^3}{(x+1)^2(x+2)^2}=\frac{A_1}{x+1}+\frac{A_2}{(x+1)^2}+\frac{A_3}{x+2}+\frac{A_4}{(x+2)^2}.....(1)$

Multiplying throughout by $(x+1)^2(x+2)^2$, we get

$x^3=A_1(x+1)(x+2)^2+A_2(x+2)^2+A_3(x+2)(x+1)^2+A_4(x+1)^2......(2)$

Substitute $x=-1,-2$ to obtain $-1=A_2$ and $-8=A_4$

Equation coefficients of x^3 on the two sides of (2), we get $1=A_1+A_3$,

Classwork Solutions-Logarithms

1. Since $\log_4 5=a \;\therefore\; 4^a=5$. Since $\log_5 6=b \;\therefore\; 5^b=6$

$\therefore\; (4^a)^b=6\Rightarrow 4^{ab}=6\Rightarrow (2^2)^{ab}=6\Rightarrow 2^{2ab}=2^1(3)\Rightarrow 2^{2ab-1}=3$

$\therefore \log_2 3=2ab-1\Rightarrow \log_3 2=\frac{1}{\log_2 3}=\frac{1}{2ab-1}$

2. $\log_3 4\log_4 5=\log_3 5$, $\log_5 6\log_6 7=\log_5 7$, $\log_7 8\log_8 9=\log_7 9$

$\therefore$ given expression $=\log_3 5\log_5 7\log_7 9=\log_3 9=\log_3 3^2=2\log_3 3=2(1)=2$

3. $\log_e\left(\frac{a+b}{2}\right)=\frac{1}{2}(\log_e a+\log_e b)=\frac{1}{2}\log_e(ab)=\log_e\left(\sqrt{ab}\right)$

$\Rightarrow\frac{a+b}{2}=\sqrt{ab}\Rightarrow a+b=2\sqrt{ab}\Rightarrow\left(\sqrt{a}-\sqrt{b}\right)^2=0\Rightarrow\sqrt{a}-\sqrt{b}=0\Rightarrow 0\Rightarrow a=b$

4. $G\cdot E\cdot=(1+\log_a bc)^{-1}+(1+\log_b ca)^{-1}+(1+\log_c ab)^{-1}$

$=(\log_a abc)^{-1}+(\log_b abc)^{-1}+(\log_c abc)^{-1}=\log_{abc}a+\log_{abc}b+\log c_{abc}=\log_{abc}abc=1$

5. $3\log\frac{81}{80}+5\log\frac{25}{24}+7\log\frac{16}{15}$

$= 3\left[\log 3^4 - \log 2^4 - \log 5\right] + 5\left[2\log 5 - \log 3 - 3\log 2\right] + 7\left[4\log 2 - \log 5 - \log 3\right]$

$= 12\log 3 - 12\log 2 - 3\log 5 + 10\log 5 - 5\log 3 - 15\log 2 + 28\log 2 - 7\log 5 - 7\log 3 = \log 2$

6. $\dfrac{\log 3}{x-y} = \dfrac{\log 5}{y-z} = \dfrac{\log 7}{z-x} = K(say)$

$\therefore\ 3 = e^{K(x-y)} \Rightarrow 3^{x+y} = e^{K\left(x^2-y^2\right)}$

Similarly $5^{y+z} = e^{K\left(y^2-z^2\right)}$

$7^{z+x} = e^{K\left(z^2-x^2\right)}$

$\therefore\ 3^{x+y}\cdot 5^{y+z}\cdot 7^{z+x} = e^{K\left(x^2-y^2+y^2-z^2+z^2-x^2\right)} = e^{K(0)} = e^0 = 1$

7. Since x, y, z are consecutive integers $\quad \therefore\ y = x+1,\ z = x+2$

Now $\log(1+zx) = \log(1+x(x+2)) = \log\left(x^2+2x+1\right) = \log(x+1)^2 = 2\log(x+1) = 2\log y$

8. $\left(2^2\right)^{\log_9 3} = 2^{2\log_9 3} = 2^{\log_9 9} = 2^1 = 2$

$9^{\log_2 4} = \left(3^2\right)^{\log_2 4} = 3^{2\log_2 4} = 3^{4\log_2 2} = 3^4 = 81$

$L\cdot H\cdot S = 2 + 81 = 83 \quad \therefore 10^{\log_x 83} = 83 \Rightarrow \log_x 83 = \log_{10} 83 \Rightarrow x = 10$

9. $\log_7\left[\log_5\left(\sqrt{x+5}+\sqrt{x}\right)\right] = 0 \Rightarrow \log_5\left(\sqrt{x+5}+\sqrt{x}\right) = 7^\circ = 1$

$\Rightarrow \sqrt{x+5}+\sqrt{x} = 5^1 = 5 \Rightarrow x+5+x+2\sqrt{x+5}\sqrt{x} = 25$

$\Rightarrow 2\sqrt{x^2+5x} = 20-2x \Rightarrow \sqrt{x^2+5x} = 10-x \Rightarrow x^2+5x = 100+x^2-20x \Rightarrow 25x = 100 \Rightarrow x = 4$

10. Given value

$= \log_{10} 2 + \log_{10} 3 + \log_{10} 5 + \log_{10} 7 + \log_{10} 11 = \log_{10}(2\cdot 3\cdot 5\cdot 7\cdot 11) = \log_{10}(2310)$

11. $x\log 2 = y\log 4 = z\log 8$

$\Rightarrow x\log 2 = y\log 2^2 = z\log 2^3 \Rightarrow x\log 2 = 2y\log 2 = 3z\log 2$

$\Rightarrow x = 2y = 3z \Rightarrow y = \dfrac{x}{2}, z = \dfrac{x}{3}$

$\therefore y+z = \dfrac{x}{2}+\dfrac{x}{3} = \dfrac{3x+2x}{6} = \dfrac{5x}{6} \qquad \therefore 5x = 6(y+z)$

12. $y = 7^{20} \Rightarrow \log y = 20\log 7 \quad \Rightarrow \log y = 20(0\cdot 8451) = 16\cdot 9020$

$\therefore$ characteristic of $\log 7^{20}$ is 16

$\therefore$ the number of digits is 7^{20} is 17

13. G. E $= \log_n 2 + \log_n 3 + \log_n 4 + \cdots + \log_n 1944$

$= \log_n(2\cdot 3\cdot 4\cdots 1944) = \log_n(1944)! = \log_n n = 1 \quad (\because n = 1944!)$

14. $(81)^{\left(\frac{1}{\log_5 3}\right)} = 3^{4\log_3 5} = 3^{\log_3\left(5^4\right)} = 5^4 = 625$

$(27)^{\log_9 36} = 3^{3\cdot 2\log_9 6} = 3^{6\cdot\log_9 6}$

$= 3^{6\frac{1}{2}\log_3 6}\left(\because \log_9 6 = \frac{1}{\log_6 9} = \frac{1}{2\log_6 3}\right) = 3^{\log_3\left(6^3\right)} = 6^3 = 216$

$3^{\frac{4}{\log_7 9}} = 3^{4\log_9 7} = 3^{4\cdot\frac{1}{2}\log_3 7} = 3^{\log_3\left(7^2\right)} = 49 \qquad \therefore$ G. E. $= 625 + 216 + 49 = 890$

15. $0\cdot 1 + 0\cdot 01 + 0\cdot 001 + \cdots \text{to } \infty = \frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \cdots = \frac{(1/10)}{1-(1/10)} = \frac{1}{9} \quad \left[s_\infty \text{ of g.p} = \frac{1}{1-r}\right]$

$G.E = \left(20^{-1}\right)^{\log_{\sqrt{20}}\left(\frac{1}{9}\right)} \quad \left(\because 0\cdot 5 = \frac{1}{20}\right) = 20^{\left(\log_{\sqrt{20}} 9\right)} \quad \left(\because -\log_{\sqrt{20}}\frac{1}{9} = \log_{\sqrt{20}} 9\right)$

$= 20^{(2\log_{20} 9)} \quad \left(\because \log_{\sqrt{20}} 9 = 2\log_{20} 9\right) = 20^{\log_{20}\left(9^2\right)} = 9^2 = 81$

16. By observation $x = 64 \quad \left(\because \log_3\left(\sqrt{64}+1\right) = \log_3 9 = 2;\ \log_2 2 = 1;\ \log_{10} 1 = 0\right)$

or $\log_{10}\log_2\log_3\left(\sqrt{x}+1\right) = 0$

$\Rightarrow \log_2\log_3\left(\sqrt{x}+1\right) = 1 \Rightarrow \log_3\left(\sqrt{x}+1\right) = 2 \Rightarrow \sqrt{x}+1 = 9 \Rightarrow \sqrt{x} = 8 \Rightarrow x = 64$

17. $x = \log_{0\cdot 1}(0\cdot 1)^3 = 3\log_{0\cdot 1}(0\cdot 1) = 3$

$y = \log_9 81 = \log_9 9^2 = 2$

$\therefore \sqrt{x - 2\sqrt{y}} = \sqrt{3 - 2\sqrt{2}} = \sqrt{(2)^2 + 1 - 2\sqrt{2}} = \sqrt{\left(\sqrt{2}-1\right)^2} = \sqrt{2} - 1$

18. Since $6^x = 7^{x+4}$

$\therefore \log 6^x = \log 7^{x+4} \Rightarrow x\log 6 = (x+4)\log 7 \Rightarrow x = [\log 6 - \log 7] = 4\log 7$

$\Rightarrow x = \frac{4\log 7}{\log 6 - \log 7} = \frac{4\log 7}{\log 2 + \log 3 - \log 7} = \frac{4c}{a+b-c}$

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Home Work Solutions:

1. $3x+4=A(x-1)-B(x-2)$

   $x=2$ gives $10=A$ $\quad x=1$ gives $7=-B(-1)=B$ $\quad \therefore (A,B)=(10,7)$

2. $A=\dfrac{1}{1+3\left(\frac{1}{2}\right)}=\dfrac{2}{5}; B=\dfrac{1}{1-2\left(-\frac{1}{5}\right)}=\dfrac{3}{5}$. Thus $2B=3A$

3. Multiply both sides by $(x+2)^3$, we get $x^2-4x-15=A(x+2)^2+B(x+2)+C$ Compare co-efficiency of various powers of x_1 on both sides

   $x^2)1=A \therefore A=1$ $\quad x)-4=4A+B=4+B \therefore B=-8$ constant term)

   $-15=4A+2B+C=4-16+C$

   $\therefore C=16-15-4=-3$

   $\therefore C=-3 \quad 2(C-A)=2(-3-1)=-8=B$

4. Multiply both sides by $(x+2)^2(x+1)$, we get $2x+3=A(x+2)^2+B(x+2)(x+1)+C(x+1)$

   Compare co-efficiency of various powers of x on both sides, we get

   $x^2)0=A+B......(i)$

   $x)2=4A+3B+C......(ii)$

   Constant term) $3=4A+2B+C.....(iii)$

   $(ii)-(iii)$ gives $A-1=0 \therefore A=1$

5. Multiplying throughout by $x^2(x-1)$, we get $1=A(x-1)x+B(x-1)+Cx^2$

   Put $x=0,1 \Rightarrow 1=-B, 1=C$ Equating coefficient of x^2 in (i),

   we get $0=A+C \Rightarrow A=-C=-1$.

6. We have $\dfrac{1}{(1-ax)^2(1+x)}=\dfrac{A}{(1-ax)^2}+\dfrac{B}{(1-ax)(1+x)}=\dfrac{A}{(1-ax)^2}+B\left\{\dfrac{A}{1-ax}+\dfrac{B}{1+x}\right\}$

   $=\dfrac{A}{(1-ax)^2}+\dfrac{AB}{1-ax}+\dfrac{B^2}{1+x}$ which are the required partical fractions.

7. Use "every where but not there" method.

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8. Multiply both sides by $(x^2+1)^2(x-1)$, we get

$x=(Ax+B)(x^2+1)(x-1)+(Cx+D)(x-1)+E(x^2+1)^2$ ......(1)

Putting $x=1$ on both sides of (1), we get $1=E(1+1)^2=4E$

$\therefore\ E=\frac{1}{4}$ Compare co-efficient of x^4 on both sides, we get $0=A+E=A+\frac{1}{4}\therefore A=-\frac{1}{4}$

$\therefore(c)$ is correct.

9. Multiply both sides by $(x^2+1)(x+1)$, we get $2x+3=(Ax+B)(x+1)+C(x^2+1)$ Putting $x=-1$

on both sides (1), we get $-2+3=0+C(1+1)=2C+D(x^2+1)$.....(1)

Putting $x=-1$ on both sides, we get $2(-1)+3=(0)+C(1+1)$

$1=C(2)\Rightarrow C=1/2\ \therefore(a)$ is correct

10. Multiply thro' out by $(x^2+2)^2$, we get $x^2+5=x^2+2+K$, $5=2+K\ \ \therefore K=3$

11. $\frac{1}{(x+a)(x^2+b^2)^2}=\frac{1}{(x^2+b^2)}\left[\frac{1}{(x+a)(x^2+b^2)}\right]$

$=\frac{1}{x^2+b^2}\left[\frac{A}{x+a}+\frac{Bx+C}{x^2+b^2}\right]=\frac{A}{(x+a)(x^2+b^2)}+\frac{Bx+C}{(x^2+b^2)^2}$

$=A\left[\frac{A}{x+a}+\frac{Bx+C}{x^2+b^2}\right]+\frac{Bx+C}{(x^2+b^2)^2}=\frac{A^2}{x+a}+\frac{ABx+AC}{x^2+b^2}+\frac{Bx+C}{(x^2+b^2)^2}$

12. Put $x^2=y$, $\therefore$ given factor $=\frac{y}{(y+a^2)(y+b^2)}=\frac{A_1}{y+a^2}+\frac{A_2}{y+b^2}$

Where $A_1=\frac{-a^2}{-a^2+b^2}=\frac{a^2}{a^2-b^2}$ $\quad A_2=\frac{-b^2}{a^2-b^2}$ $\quad \therefore A=\frac{a^2}{a^2-b^2}, B=0$

13. Putting $x^2=y$ and using "everywhere but not there methods",

$A=\frac{-5+3}{(-5-7)(-5+9)}=\frac{-2}{-48}=\frac{1}{24}$

$B=\frac{7+3}{(7+5)(7+9)}=\frac{10}{192}=\frac{5}{96}$ and $C=\frac{-9+3}{(-9+5)(-9-7)}=\frac{-6}{(-4)(-16)}=-\frac{3}{32}$

14. $2x=A(x^2+x+1)+B(x^2-x+1)$

Coefficient of x^2, $0=A+B$, Coefficient of x, $2=A-B\quad\Rightarrow 2A=2$

$\Rightarrow A=1 \quad 2B=-2 \qquad \Rightarrow B=-1 \qquad \therefore AB=-1$

15. $1+abc=1+\log_{24}12\log_{36}24\log_{48}36$

$=1+\log_{48}12=\log_{48}48+\log_{48}12=\log_{48}(48\times 12)$

$=\log_{48}\left(24^2\right)=2\log_{48}24=2\log_{48}36\log_{36}24=2cb=2bc$

16. $\log_7 2=m$

$\log_{49}28=\frac{\log 28}{\log 49}=\frac{\log 7+\log 4}{2\log 7}=\frac{\log 7}{2\log 7}+\frac{\log 4}{2\log 7}=\frac{1}{2}+\frac{1}{2}\log_7 4=\frac{1}{2}+\frac{1}{2}\cdot 2\log_7 2$

$=\frac{1}{2}+\log_7 2=\frac{1}{2}+m=\frac{1+2m}{2}$

17. $\log_5 a\cdot\log_a x=2\Rightarrow\log_5 x=2\Rightarrow 5^2=x$ i.e., $x=25$

18. Clearly $x=e^{K\left(a^2+ab+b^2\right)}$

$\therefore x^{a-b}=e^{K\left(a^3-b^3\right)}etc \qquad \therefore x^{a-b}\cdot y^{b-c}\cdot z^{c-a}=e^0=1$

19. By the given result,

$2\log 2+\log a+\log b=2\log(a+b)$

$\Rightarrow\log(4ab)=\log(a+b)^2\Rightarrow(a+b)^2=4ab\Rightarrow(a-b)^2=0\Rightarrow a=b$

20. $\log_4 2+\log_4 4+\log_4 16+\log_4 x=6\Rightarrow\log_4(2\times 4\times 16\times x)=6$

$\log_4(128x)=6\Rightarrow 128x=4^6=4\times 4\times 4\times 4\times 4\times 4$

$\Rightarrow x=\frac{4\times 4\times 4\times 4\times 4\times 4}{128}=2\times 4\times 4=32$ $\therefore$ (c) holds

21. Given result $\Rightarrow\log_x a+\log_x b+\log_x c=0\Rightarrow\log_x(abc)=0\Rightarrow abc=x^{\circ}=1$

22. Given result $=2\log_y\cdot x\cdot 3\log_z y\cdot 4\log_x z=24\left[\log_y x\cdot\log_z y\cdot\log_x z\right]=24\log_x x=24\cdot 1=24$

23. Given result

$=\log x-\log(x+1)+\log(x+1)-\log(x+2)+\log(x+2)-\log(x+3)+\cdots+\log(x+n-1)-\log(x+n)$

$=\log x-\log(x+n)=\log\frac{x}{x+n}$

24. $\log_{30}15=\log_{30}3+\log_{30}5=a+b$

$\log_{30}8=\log_{30}2^3=\log_2 2^3\cdot\log_{30}2$

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$= \log_2 2^3 \cdot \log_{30}\left(\frac{30}{15}\right) = 3\log_2 2\left[\log_{30} 30 - \log_{30} 15\right] = 3\left[1 - a - b\right]$

25. Given result $= \dfrac{\frac{1}{\log_x a} \times \frac{1}{\log_x b}}{\frac{1}{\log_x a} + \frac{1}{\log_x b}} = \dfrac{1}{\log_x b + \log_x a} = \dfrac{1}{\log_x (ab)} = \log_{ab}(x)$

26. $y = (0 \cdot 5)^{50} \Rightarrow \log y = 50\log(0 \cdot 5)$

$\Rightarrow \log y = 50\left(\bar{1} \cdot 6900\right) \Rightarrow \log y = 50(-1 + 0 \cdot 6990)$

$\Rightarrow \log y = -50 + 34 \cdot 9500 \Rightarrow \log y = -16 + 0 \cdot 9500 = \overline{16} \cdot 9500$

$\therefore$ Characteristic is $\overline{16} \Rightarrow$ Number of zeros is 16 – 1 = 15

27. $\log_3 5 \cdot \log_{25} 27 = \log_3 5 \cdot 3\log_{25} 3 = 3 \cdot \log_3 5 \cdot \dfrac{1}{2\log_3 5} = \dfrac{3}{2}$

28. Consider, $\log_{0.5} 4 = \log_{1/2} 4 = \dfrac{\log 4}{\log\frac{1}{2}} = \dfrac{2\log 2}{-\log 2}\left(\because \log\frac{1}{2} = -\log 2\right) = -2$

29. $\log_4(\log_3 x) = \dfrac{1}{2} \Rightarrow \log_3 x = 2 \Rightarrow x = 3^2 = 9$

30. $\log_9 x + \log_3 x = 6 \Rightarrow \dfrac{1}{\log_x 9} + \dfrac{1}{\log_x 3} = 6 \Rightarrow \dfrac{1}{2} \cdot \dfrac{1}{\log_x 3} + \dfrac{1}{\log_x 3} = 6$

$\Rightarrow \dfrac{3}{2}\dfrac{1}{\log_x 3} = 6 \Rightarrow \dfrac{1}{\log_x 3} = 4 \Rightarrow \log_3 x = 4 \Rightarrow x = 3^4 = 81$