

Subject	Topic	Lecture No.
Mathematics	Partial Fraction, Logarithm	M-01

## Partial Fraction

An expression of the form  $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_n$  where  $n$  is a positive integer and  $a_0, a_1, a_2, \dots, a_n$  are real numbers is called a polynomial of degree  $n$  in the variable  $x$ .

## Rational Functions

If  $f(x)$  and  $g(x)$  are two polynomials in  $x$  and  $g(x) \neq 0$  then  $\frac{f(x)}{g(x)}$  is called Rational function

If degree of  $f(x)$  is less than the degree of  $g(x)$  then the rational function is known as proper fraction.

If degree of  $f(x)$  is greater than or equal to the degree of  $g(x)$ , then the rational function is known as Improper fraction. Ex:  $\frac{3x-7}{x^3-x}$  is a proper fraction,  $\frac{x^2+9x+19}{x^2+7x+12}$ ,  $\frac{3-4x-5x^2-x^3}{(x+3)(x+2)}$  are improper fractions.

## Rules to resolve a rational function into partial fraction

**Rule (i)** : To each non-repeated linear factor  $ax+b$  of  $g(x)$ , there corresponds a partial fraction of the form  $\frac{A}{ax+b}$ , where  $A$  is a constant  $\neq 0$ .

**Rule (ii)** : To each repeated linear factor  $(ax+b)^r$  of  $g(x)$ , there corresponds  $r$  partial fractions of the form  $\frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \dots + \frac{A_r}{(ax+b)^r}$ , where  $A_1, A_2, \dots, A_r$  are constants,  $A_r \neq 0$ .

**Rule (iii)** : To each non-repeated irreducible quadratic factor  $ax^2+bx+c$  of  $g(x)$  there corresponds a partial fraction of the form  $\frac{Ax+B}{ax^2+bx+c}$  where  $A$  and  $B$  are constants not both zero.

## Time saving results (Short cut methods)

- If  $\frac{f(x)}{(x-a)(x-b)}$  is a proper rational function and  $\frac{f(x)}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$  then  $A = \frac{f(a)}{a-b}$ ,  $B = \frac{f(b)}{b-a}$ .
- $\frac{1}{(x-a)(x-b)} = \frac{1}{a-b} \left[ \frac{1}{x-a} - \frac{1}{x-b} \right]$
- If  $\frac{Px+Q}{(x-a)(x-b)^2} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{(x-b)^2}$  then  $A = \frac{Pa^2+Qa+R}{(a-b)^2}$ ,  $B = P-A$ ,  $C = \frac{Pb^2+Qb+R}{b-a}$ .
- If  $\frac{Px+Q}{x(x^2+a^2)} = \frac{A}{x} + \frac{Bx+C}{x^2+a^2}$  then  $A = \frac{Q}{a^2}$ ,  $B = -A$ ,  $C = P - \frac{Q}{a^2}$  = coefficient of  $x$  in the numerator.
- If  $\frac{Px+Q}{(x-a)(x^2+b^2)} = \frac{A}{x-a} + \frac{Bx+C}{x^2+b^2}$  then  $A = \frac{Pa+Q}{a^2+b^2}$ ,  $B = -A$ ,  $C = P - aA$ .





Subject	Topic	Lecture No.
Mathematics	Partial Fraction, Logarithm	M-01

5. If  $\frac{1}{(1-ax)(1-bx)} = \frac{A}{1-ax} + \frac{B}{1-bx}$ , then  $\frac{A}{(1-ax)^2(1-bx)} =$
- (a)  $\frac{A}{1-ax} + \frac{B}{(1-bx)^2} + \frac{AB}{(1-ax)(1-bx)}$       (b)  $\frac{A}{(1-ax)^2} + \frac{AB}{1-ax} + \frac{B^2}{1-bx}$
- (c)  $\frac{A}{1-ax} + \frac{AB}{(1-ax)(1-bx)}$       (d)  $\frac{A}{(1-ax)^2} + \frac{B}{1-bx}$
6. If  $\frac{3x+4}{(x+1)^2(x-1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$ , then  $A =$
- (a)  $-\frac{1}{2}$       (b)  $\frac{15}{4}$       (c)  $\frac{7}{4}$       (d)  $-\frac{1}{4}$
7. If  $\frac{1-x+6x^2}{x-x^3} = \frac{A}{x} + \frac{B}{1-x} + \frac{C}{1+x}$ , then  $A =$
- (a) 1      (b) 2      (c) 3      (d) 4
8. Let  $a, b, c$  such that  $\frac{1}{(1-x)(1-2x)(1-3x)} = \frac{a}{1-x} + \frac{b}{1-2x} + \frac{c}{1-3x}$ , then  $\frac{a}{1} + \frac{b}{3} + \frac{c}{5} =$
- (a)  $\frac{1}{15}$       (b)  $\frac{1}{6}$       (c)  $\frac{1}{5}$       (d)  $\frac{1}{3}$
9. If  $\frac{(1+x)(1+2x)(1+3x)}{(1-2x)(1-3x)(1-4x)} = Q + \frac{A}{1-2x} + \frac{B}{1-3x} + \frac{C}{1-4x}$ , where  $Q, A, B, C \in R$ , then  $Q =$
- (a)  $\frac{1}{4}$       (b) 4      (c)  $-\frac{1}{4}$       (d) -4
10.  $\frac{2x^2+3x+4}{(x-1)(x^2+2)} =$
- (a)  $\frac{3}{x-1} - \frac{x-2}{x^2+2}$       (b)  $\frac{4}{x-1} - \frac{x-2}{x^2+2}$       (c)  $\frac{1}{x-1} + \frac{x-2}{x^2+2}$       (d) none of these
11. If  $\frac{x^2}{(x^2+a^2)(x^2+b^2)} = K \left( \frac{a^2}{x^2+a^2} - \frac{a^2}{x^2+b^2} \right)$ , then  $K =$
- (a)  $a^2 - b^2$       (b)  $\frac{1}{a+b}$       (c)  $\frac{1}{a-b}$       (d)  $\frac{1}{a^2 - b^2}$
12. Remainder of  $x^{64} + x^{27} + 1$  divided by  $x+1$  is
- (a) 0      (b) 1      (c) 2      (d) 3
13. If  $\frac{(x+1)^2}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$ , then  $A+B+C =$

Subject	Topic	Lecture No.
Mathematics	Partial Fraction, Logarithm	M-01

- (a) -4                      (b) 2                      (c) 4                      (d) none of these

14. When resolved into partial fractions,  $\frac{x^2 + 1}{x^4 + x^2 + 1} =$

- (a)  $\frac{Ax + B}{x^2 + x + 1} + \frac{Cx + D}{x^2 - x + 1}$                       (b)  $\frac{Ax + B}{x^2 + \sqrt{x} + 1} + \frac{Cx + D}{x^2 - \sqrt{x} + 1}$   
(c)  $\frac{Ax + B}{x^2 + \sqrt{x} + 2} + \frac{Cx + D}{x^2 - \sqrt{x} + 2}$                       (d) none of these

15. While resolving  $\frac{3x^2 - 4x + 5}{(x^2 - x + 1)^3}$ , the number of partial fractions are

- (a) 2                      (b) 3                      (c) 4                      (d) none of these

16. If  $\frac{x^3}{(x+1)^2(x+2)^2} = \frac{A_1}{x+1} + \frac{A_2}{(x+1)^2} + \frac{A_3}{x+2} + \frac{A_4}{(x+2)^2}$ , then

- (a)  $A_2 = -1, A_4 = -8$                       (b)  $A_1 + A_3 = 4$                       (c)  $A_2 = 1, A_4 = 8$                       (d) none of these

**Partial fractions (Class work answer)**

1. c	2. a	3. c	4. a	5. b	6. c	7. a	8. a	9. c	10. a
11. d	12. b	13. d	14. a	15. b	16. a				

**Logarithms**

If  $a$  is a positive real number  $\neq 1$  and  $y = a^x$  then  $x$  is called the logarithm of  $y$  to the base  $a$  and is denoted by  $x = \log_a y$ .

**Laws of logarithm**

1.  $\log_a uv = \log_a u + \log_a v$
2.  $\log_a \left(\frac{u}{v}\right) = \log_a u - \log_a v$
3.  $\log_a u^p = p \log_a u$
4.  $\log_b u = \frac{\log_a u}{\log_a b}$  [change of base rule]
5.  $\log_{a^p} u = \frac{1}{p} \log_a u$
6.  $a^{\log_a u} = u$
7. The base of a logarithm must always be positive.
8.  $\log_a 0$  is not defined and we take  $\log_a 0 = -\infty$  ( $\because a^{-\infty} = 0$ )

Subject	Topic	Lecture No.
Mathematics	Partial Fraction, Logarithm	M-01

Since  $a^0 = 1$ , for all real numbers  $a \neq 0$ ,  $\log_a 1 = 0$ . Thus logarithm of 1 to any base is zero.

Since  $a^1 = a$ ,  $\log_a a = 1$ , i.e. logarithm of any number to the same base is equal to 1

**Time saving results (Short cut methods)**

- $\log_a b \cdot \log_c a = \log_c b$
- $\log_b a = \frac{1}{\log_a b}$ ,  $\log_a a^2 = 2$ ,  $\log_{a^2} a = \frac{1}{2}$
- $\log_{b^p} a = \frac{1}{p} \log_b a$ ,  $\log_{a^2} (a^p) = \frac{p}{2}$ ,  $\log_{b^q} a^p = \frac{p}{q} \log_b a$
- If  $\log y = m$  (.....) then the number of digits is  $(m+1)$ .
- If  $\log y = \bar{m}$  (.....) then the number of zeros before the decimal in  $y$  is  $(m+1)$ .

**Class Work Problems**

1. If  $\log_4 5 = a$  and  $\log_5 6 = b$ , then  $\log_3 2$  is
 

(a) $\frac{1}{2a+1}$	(b) $\frac{1}{2b+1}$	(c) $2ab+1$	(d) $\frac{1}{2ab-1}$
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2. The value of  $\log_3 4 \log_4 5 \log_5 6 \log_6 7 \log_7 8 \log_8 9$  is
 

(a) 1	(b) 2	(c) 3	(d) 4
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3. If  $\log_e \left( \frac{a+b}{2} \right) = \frac{1}{2} (\log_e a + \log_e b)$ , then relation between  $a$  and  $b$  will be
 

(a) $a = b$	(b) $a = \frac{b}{2}$	(c) $2a = b$	(d) $a = \frac{b}{3}$
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4. If  $x = \log_a (bc)$ ,  $y = \log_b (ca)$ ,  $z = \log_c (ab)$ , then which of the following is equal to 1?
 

(a) $x + y + z$	(b) $(1+x)^{-1} + (1+y)^{-1} + (1+z)^{-1}$
(c) $xyz$	(d) none of these
5. The value of  $3 \log \frac{81}{80} + 5 \log \frac{25}{24} + 7 \log \frac{16}{15}$  is
 

(a) $\log 2$	(b) $\log 3$	(c) 1	(d) 0
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6. If  $\frac{\log 3}{x-y} = \frac{\log 5}{y-z} = \frac{\log 7}{z-x}$ , then  $3^{x+y} \cdot 5^{y+z} \cdot 7^{z+x} =$ 

(a) 0	(b) 2	(c) 1	(d) none of these
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7. If  $x, y, z$  are three consecutive positive integers, then  $\log(1+zx) =$ 

(a) $\log y$	(b) $\log \frac{y}{2}$	(c) $\log(2y)$	(d) $2 \log y$
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8. If  $4^{\log_9 3} + 9^{\log_2 4} = 10^{\log_x 83}$ , then  $x =$

Subject	Topic	Lecture No.
Mathematics	Partial Fraction, Logarithm	M-01

- (a) 5                      (b)  $\frac{1}{10}$                       (c) 10                      (d) 25

9. If  $\log_7(\log_5(\sqrt{x+5} + \sqrt{x})) = 0$ , then the value of  $x$  is

- (a) 3                      (b) 5                      (c) 1                      (d) 4

10. The value of  $\frac{1}{\log_2 10} + \frac{1}{\log_3 10} + \frac{1}{\log_5 10} + \frac{1}{\log_7 10} + \frac{1}{\log_{11} 10}$  is

- (a)  $\log_{10}(420)$                       (b)  $\log_{10}(2310)$                       (c)  $\log_{10}(11)$                       (d) none of these

11. If  $x \log 2 = y \log 4 = z \log 8$ , then

- (a)  $x = 6(y + z)$                       (b)  $6x = y + z$                       (c)  $5x = 6(y + z)$                       (d)  $5(x + y) = 6z$

12. If  $\log 7 = 0.8451$ , then number of digits in  $7^{20}$  is

- (a) 16                      (b) 17                      (c) 18                      (d) 20

13. If  $n = (1944)!$  then the sum  $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \dots + \frac{1}{\log_{1944} n} =$

- (a) 0                      (b) 1                      (c)  $\frac{1}{2}$                       (d) 2

14.  $81^{\left(\frac{1}{\log_5 3}\right)} + 27^{(\log_9 36)} + 3^{\left(\frac{4}{\log_7 9}\right)} =$

- (a) 49                      (b) 625                      (c) 216                      (d) 890

15.  $(0.05)^{\log_{\sqrt{20}}(0.1+0.01+0.001+\dots)} =$

- (a) 81                      (b)  $\frac{1}{81}$                       (c) 20                      (d)  $\frac{1}{20}$

16. If  $\log_{10} \log_2 \log_3(\sqrt{x} + 1) = 0$ , then  $x =$

- (a) 16                      (b) 9                      (c) 49                      (d) 64

17. If  $x = \log_{0.1}(0.001)$ ,  $y = \log_9 81$ , then  $\sqrt{x - 2\sqrt{y}} =$

- (a)  $3 - 2\sqrt{2}$                       (b)  $\sqrt{3} - 2$                       (c)  $\sqrt{2} - 1$                       (d)  $\sqrt{2} - 2$

18. If  $\log 2 = a, \log 3 = b, \log 7 = c$  and  $6^x = 7^{x+4}$ , then  $x =$

- (a)  $\frac{4b}{b+a-c}$                       (b)  $\frac{4c}{a+b-c}$                       (c)  $\frac{4b}{c-a-b}$                       (d)  $\frac{4a}{a+b-c}$

Subject	Topic	Lecture No.
Mathematics	Partial Fraction, Logarithm	M-01

**Logarithm Class work Answers**

1. d	2. b	3. a	4. b	5. a	6. c	7. d	8. c	9. d	10. b
11. c	12. b	13. b	14. d	15. a	16. d	17. c	18. b		

**Homework problems**

1. If  $\frac{3x+4}{x^2-3x+2} = \frac{A}{x-2} - \frac{B}{x-1}$  then  $(A, B)$

- (a) (7,10)                      (b) (10,7)                      (c) (10,-7)                      (d) (-10,7)

2. If  $\frac{1}{(1-2x)(1+3x)} = \frac{A}{1-2x} + \frac{B}{1+3x}$  then  $2B =$

- (a)  $A$                               (b)  $2A$                               (c)  $3A$                               (d)  $-3A$

3. If  $\frac{x^2-4x-15}{(x+2)^3} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{(x+2)^3}$ , then

- (a)  $B = 2(C - A)$               (b)  $A - B + C = 0$               (c)  $A = B = 2C$               (d) none of these

4. If  $\frac{2x+3}{(x+2)^2(x+1)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$  then  $A =$

- (a)  $-1$                               (b)  $1$                               (c)  $5$                               (d)  $-5$

5. If  $\frac{1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$ , then

- (a)  $A = -1$                       (b)  $B = 1$                       (c)  $C = -1$                       (d) none of these

6. If  $\frac{1}{(1-ax)(1+x)} = \frac{A}{1-ax} + \frac{B}{1+x}$  then

(a)  $\frac{1}{(1-ax)^2(1+x)} = \frac{A^2}{(1-ax)^2} + \frac{AB}{(1-ax)} + \frac{B}{1+x}$

(b)  $\frac{1}{(1-ax)^2(1+x)} = \frac{A}{(1-ax)^2} + \frac{AB}{1-ax} + \frac{B^2}{1+x}$

(c)  $\frac{1}{(1-ax)^2(1+x)} = \frac{A^2}{(1-ax)^2} + \frac{A}{1-ax} + \frac{B^2}{1+x}$

- (d) none of these

7. If  $\frac{x^2}{(x-a)(x-b)(x-c)} = \frac{P}{x-a} + \frac{Q}{x-b} + \frac{R}{x-c}$

- (a)  $P = \frac{a^2}{(a-b)(a-c)}$       (b)  $Q = \frac{b^2}{(b+a)(b+c)}$       (c)  $R = \frac{c^2}{(c+a)(c+b)}$       (d) none of these



Subject	Topic	Lecture No.
Mathematics	Partial Fraction, Logarithm	M-01

8. If  $\frac{x}{(x^2+1)^2(x-1)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2} + \frac{E}{x-1}$ , then  $A =$
- (a)  $\frac{1}{4}$                       (b)  $-\frac{1}{3}$                       (c)  $-\frac{1}{4}$                       (d)  $-\frac{1}{5}$
9. If  $\frac{2x+3}{(x^2+1)(x+1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1}$  then  $C =$
- (a)  $\frac{1}{2}$                       (b)  $-\frac{1}{2}$                       (c)  $\frac{1}{3}$                       (d)  $-\frac{1}{3}$
10. If  $\frac{x^2+5}{(x^2+2)^2} = \frac{1}{x^2+2} + \frac{K}{(x^2+2)^2}$ , then  $K =$
- (a) 1                      (b) 2                      (c) 3                      (d) 4
11. If  $\frac{1}{(x+a)(x^2+b^2)} = \frac{A}{x+a} + \frac{Bx+C}{x^2+b^2}$ , then  $\frac{1}{(x+a)(x^2+b^2)^2} =$
- (a)  $\frac{A^2}{x+a} + \frac{ABx+AC}{x^2+b^2} + \frac{Bx+C}{(x^2+b^2)^2}$                       (b)  $\frac{A^2}{x+a} + \frac{ABx}{x^2+b^2} + \frac{Bx+C}{(x^2+b^2)^2}$
- (c)  $\frac{A^2}{x+a} + \frac{AC}{x^2+b^2} + \frac{Bx+C}{(x^2+b^2)^2}$                       (d) none of these
12. If  $\frac{x^2}{(x^2+a^2)(x^2+b^2)} = \frac{Ax+B}{x^2+a^2} + \frac{Cx+D}{x^2+b^2}$  then  $(A, B)$  are
- (a)  $\left(\frac{a^2}{a^2-b^2}, 0\right)$                       (b)  $\left(\frac{b^2}{a^2-b^2}, 0\right)$                       (c)  $\left(\frac{a^2}{a^2+b^2}, 0\right)$                       (d)  $\left(\frac{b^2}{a^2+b^2}, 0\right)$
13. If  $\frac{x^2+3}{(x^2+5)(x^2-7)(x^2+9)} = \frac{A}{x^2+5} + \frac{B}{x^2-7} + \frac{C}{x^2+9}$ , then
- (a)  $A = \frac{1}{24}$                       (b)  $B = -\frac{5}{96}$                       (c)  $C = \frac{3}{32}$                       (d) none of these
14. If  $\frac{2x}{x^4+x^2+1} = \frac{A}{x^2-x+1} + \frac{B}{x^2+x+1}$  then  $AB =$
- (a) -1                      (b)  $\frac{1}{2}$                       (c) 1                      (d) 2
15. If  $a = \log_{24} 12$ ,  $b = \log_{36} 24$  and  $c = \log_{48} 36$ , then  $1+abc$  is equal to
- (a)  $2ab$                       (b)  $2ac$                       (c)  $2bc$                       (d) 0
16. If  $\log_7 2 = m$ , then  $\log_{49} 28$  is equal to
- (a)  $2(1+2m)$                       (b)  $\frac{1+2m}{2}$                       (c)  $\frac{2}{1+2m}$                       (d)  $1+m$



Subject	Topic	Lecture No.
Mathematics	Partial Fraction, Logarithm	M-01

17. If  $\log_5 a \cdot \log_a x = 2$ , then  $x$  is equal to
- (a) 125                      (b)  $a^2$                       (c) 25                      (d) none of these
18. If  $\frac{\log x}{a^2 + ab + b^2} = \frac{\log y}{b^2 + bc + c^2} = \frac{\log z}{c^2 + ac + a^2}$  then  $x^{a-b} \cdot y^{b-c} \cdot z^{c-a} =$
- (a) 0                      (b) -1                      (c) 1                      (d) 2
19. If  $\log 2 + \frac{1}{2} \log a + \frac{1}{2} \log b = \log(a+b)$ , then
- (a)  $a = b$                       (b)  $a = -b$                       (c)  $a = 2, b = 0$                       (d)  $a = 10, b = 1$
20. If  $\log_4 2 + \log_4 4 + \log_4 16 + \log_4 x = 6$ , then  $x =$
- (a) 4                      (b) 64                      (c) 32                      (d) 8
21. If  $\frac{1}{\log_a x} + \frac{1}{\log_b x} + \frac{1}{\log_c x} = 0$ , then  $abc =$
- (a) -1                      (b) 1                      (c) 2                      (d) none of these
22. The value of  $\log_y x^2 \cdot \log_z y^3 \cdot \log_x z^4$  is
- (a) 1                      (b) 12                      (c) 24                      (d) none of these
23.  $\log\left(\frac{x}{x+1}\right) + \log\left(\frac{x+1}{x+2}\right) + \log\left(\frac{x+2}{x+3}\right) \dots$  to  $n$  terms =
- (a)  $\log(x+n)$                       (b)  $\log\left(\frac{x}{x+n}\right)$                       (c)  $\log\left(\frac{x+n}{x}\right)$                       (d) none of these
24. If  $\log_{30} 3 = a, \log_{30} 5 = b$ , then  $\log_{30} 8 =$
- (a)  $3(1-a-b)$                       (b)  $3(1+a+b)$                       (c)  $3(-a+b)$                       (d)  $a+b$
25.  $\frac{\log_a x \times \log_b x}{\log_a x + \log_b x} =$
- (a)  $\log_b a$                       (b)  $\log_a x$                       (c)  $\log_{ab} x$                       (d)  $\log_x(ab)$

Subject	Topic	Lecture No.
Mathematics	Partial Fraction, Logarithm	M-01

26.  $\log 5 = 0.6990$ , the number of zeros between the decimal point and the first significant figure in  $(0.5)^{50}$  is

(a) 14                      (b) 15                      (c) 16                      (d) 17

27.  $\log_3 5 \cdot \log_{25} 27 =$

(a)  $\frac{2}{3}$                       (b)  $\frac{3}{2}$                       (c)  $-\frac{3}{2}$                       (d)  $-\frac{2}{3}$

28.  $\sqrt{(\log_{0.5} 4)^2} =$

(a) -2                      (b)  $\sqrt{-4}$                       (c) 2                      (d) none of these

29. If  $\log_4 (\log_3 x) = \frac{1}{2}$  then  $x =$

(a) 9                      (b) 6                      (c) 4                      (d) 2

30. If  $\log_9 x + \log_3 x = 6$  then  $x =$

(a) 81                      (b) 9                      (c) 3                      (d) 27

**Home Work Answer keys**

1. b	2. c	3. a	4. b	5. a	6. b	7. a	8. c	9. a	10. c
11. a	12. a	13. a	14. a	15. c	16. b	17. c	18. c	19. a	20. c
21. b	22. c	23. b	24. a	25. c	26. b	27. b	28. a	29. a	30. a